Acting and Interacting Like Me? A Method for Identifying Similarity and Synchronous Behavior between a Human and a Robot

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Abstract— This paper introduces a similarity identification method using an Information Distance methodology. We demonstrate that this method can successfully identify the similarity and synchronicity of behavior between a human and a robot. We suggest that the application of appropriate binning strategies is the key factor that drives the effectiveness of this method. Experiments are carried out that initially validate the method on simulated data and then subsequently use real-world imitation game data. The results indicate that the method is able to correctly identify both perfectly synchronous and perfectly asynchronous imitating actions.

1 Introduction

In order to exploit the opportunities that robots may offer in our daily lives, Human-Robot Interaction (HRI) has become an important topic [1]. A major research area in HRI is imitation behavior between humans and robots. A robot imitating a human may learn new skills, but also be able to engage more effectively in social interaction. Thus, a significant amount of effort has been devoted to this research topic (see, for example, [2, 3, 4, 5, 6, 7]) building on previous research in developmental psychology (such as facial imitation in infants and neonates [8]). Our current research focuses on preparatory works required to e.g. replicate human-infant experiments on the “like me” problem (see [9, 10, 11]).

In this paper we report on studies carried out which enable robots to identify similarity and synchrony between their actions and human actions. For example, a robot and human both waving their hands would indicate similarity of action, both waving in a mirror-like way would indicate synchronicity. We consider this work to be a stepping stone towards enabling a robot to learn socially from interaction with people. Being able to identify similarity and synchronicity (including when both human and robot actions are similar and perfectly asynchronous i.e. mirrored but perfectly out-of-phase) is important in allowing the robot to recognize human actions which are matching its own. It has been suggested that the identification of ‘like me’ in interaction may not only represent a salient event in the social development of an infant (cf. [8]), but, from the perspective of social robots [9, 10], may enable a robot to engage in 'meaningful' interactions with its social environment as a key ingredient of learning in a social context. A method for identifying these similar and synchronous actions is described here[1]. While the method is not directly based on neurobiological modeling, we nevertheless employ a technique using computational principles that have been shown to model the perception-action loop of an agent acting in its environment in the language of information [12]. Thus, the approach is biologically inspired, but not on the level of neurons but on the more abstract level of information. The method employs the idea of similarity using *information distance*, previously described by Crutchfield [13] and based on *information theory* [14]. Information distance is used here to capture the spatial and temporal relationships between events. Relevant research using the information distance methodology as applied and further developed in developmental robotics in our research group has been described in, for example, [15, 16, 17]. In order to be consistent with this particular research approach, we utilize the same method but apply it to a different context, namely to particularly identify similarity and synchronicity instead of using it as a general correlation between sensor data. The experimental results suggest that this method can successfully identify similar and synchronous actions in human and robot imitation behavior.

This paper will explain the similarity identification method in section 2. In section 3 initial validation experiments using this method are described followed by actual experiments on a robot platform. In Section 4 the experimental results are analyzed and we discuss these results and future work in section 5.

2 Similarity Identification Using Information Distance

The similarity identification method introduced here calculates the information distance between human and robot body part trajectories to yield an indication of their similarity. The numeric size of the information distance value gives an indication of similarity, thus the more similar the behaviors, the lower the value. Similarly, a higher value for information distance indicates less similar behaviors.

The flow chart in Figure 1 shows the general approach of the similarity method. In this flow chart, circles and ellipses represent data components; rectangles with solid lines represent core processing components and rectangles with dashed lines represent optional processing components.

The general approach of this similarity method involves three stages: data collection, which consists of the first three components in the flow chart; pre-processing, which consists of the middle four components; and the information distance calculation, which consists of the last two components. These stages will now be described in more detail below.

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[1] Note, our intention is not to propose a new method that outperforms others, but to demonstrate that a method based on information distance is suitable for the task of behaviour similarity detection, an approach that we are also using for other tasks in our computational robot control architectures.

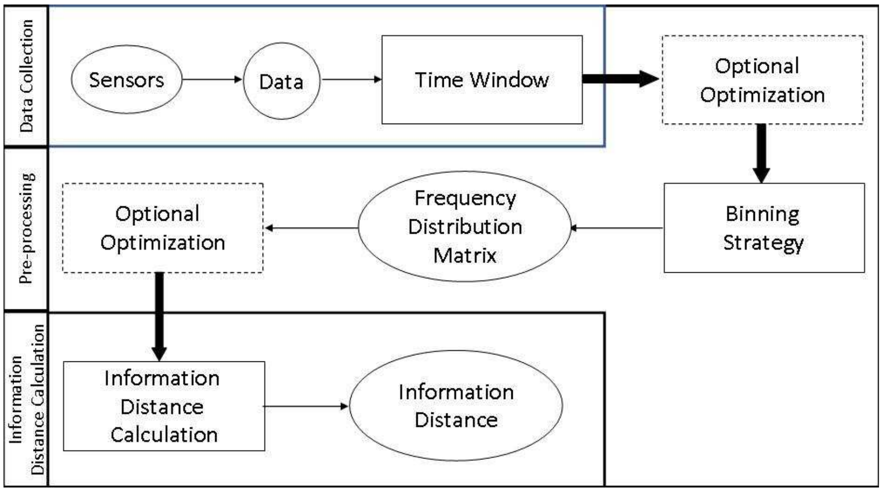

Fig. 1 The Similarity Method General Approach Flow Chart

2.1 Data Collection

In the data collection stage, a time window is used to store body parts' trajectory data of both the human and robot captured from sensors (including the internal states of the robot). For every time step, the time window is updated with the latest trajectory data collected.

The time window is a two dimensional array. One dimension is the number of time steps of the trajectory that the window can keep (treated as a row). The other dimension is the number of data items that are being tracked (treated as column). For example, if the spatial data currently being tracked is the 3-D co-ordinates of the hand position of both robot and human experimenter (x, y, z co-ordinates of the robot hand position and x, y, z co-ordinates of the human hand position) and the trajectory that is being kept is the most recent 50 time steps then a 50*6 array is allocated as the size of the time window. The size of the time window is fixed once allocated and uses a First-In- First-Out buffer to store new sensory data as it is recorded. Therefore, for each time step, the data at the back end of the window will be considered out of data and disposed of, with newly updated data added to the front end of the time window.

2.2 Binning Strategy

The data in the time window will be allocated into different bins according to its value and the binning strategy. Note that not all the data will be pre-processed at the same time. Every time the pre-process procedure is called, only two selected data columns are used. Similarly, every time the information distance calculation procedure is called, only two selected data columns are used. This is because the information distance can only be calculated between two items.

The binning strategy component is used to extract data distribution features. These features are recorded using a frequency distribution matrix and two bin frequency distribution arrays, which will be described below. They are the critical source of information to conduct the information distance calculation.

The bin frequency distribution matrix tracks how many times data items of bin x in column A appear together with data items of bin y in column B. The bin frequency distribution arrays track the number of times data items of each bin in their own column have appeared.

The two new binning strategies used in this similarity identification method, which we call *Partial-Adaptive Binning Strategy* and *Complete-Adaptive Binning Strategy* are both developed from the binning strategies described by Olsson [15], *Static Binning Strategy* and *Adaptive Binning Strategy*. However, they have significant differences due to the nature of the data in our research. In Olsson's work, the data represent pixel values of a robot's vision system, which have similar inputs. However, in the studies presented here, the input data are from different sources and may derive from different modalities. Therefore, there may be large variances in the data captured. Using the original binning strategies may cause a loss of a significant amount of information.

The newly developed binning strategies have three common factors: 'column-based independence', 'adaptive bin ranges' and 'tendency separation'. 'Column-based independence' means each column has an independent bin range. 'Adaptive bin range' means the bin range is determined by the maximum and minimum data entry within the same column. These two features cater for the fact that different columns contain data from different sensors and the range of their data values may have significant differences. Therefore, the features of different columns may be omitted if all the columns use the same bin range. 'Tendency separation' means the tendency of a data item (i.e. whether the next data item in the same column has a larger or smaller value than the current one) is considered in the bin allocation process. Practically, each bin is split into two bins: a rising bin and a descending bin. Once a data item is allocated into a bin, the tendency of this data item is examined. If the tendency is rising or staying still, the data is assigned to the rising bin. Otherwise, it will be assigned to the descending bin. Tendency separation is used to reduce the impact of the delay (or time-shift) between one agent imitating another's behavior. For example, there might be a slight delay between a human copying the actions of a robot, or vice-versa.

An example of time shift impact is presented in Figure 2. Curve A and curve B are identical except curve B is slightly shifted. Although point *a* and point *b* on curve B have the same value, the difference between their corresponding points (*c* and *d*) on curve A is significant. If only data value is considered, point *a* and point *b* will be allocated to the same bin. However, the bins that *a* and *b* belong to have the same chance of corresponding to the two bins that *c* and *d* belong to. Consequently, this one-to-many relationship causes an ambiguity and omits the fact that there is one-to-one relationship existing if the slope factor is considered. Figure 3 shows a robot and human forearm X-axis trajectory (where a human was attempting to replicate a robot movement) and illustrates the existence of this time shift impact in real life. During the imitation interaction, it is almost impossible to synchronize robot and human behavior perfectly. There are always some differences in timing between the two behaviors.

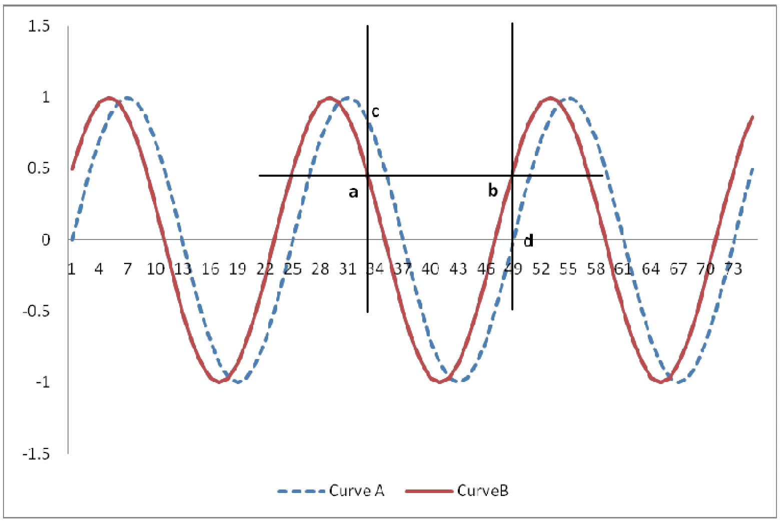

Fig. 2 Time Shift Impact Example

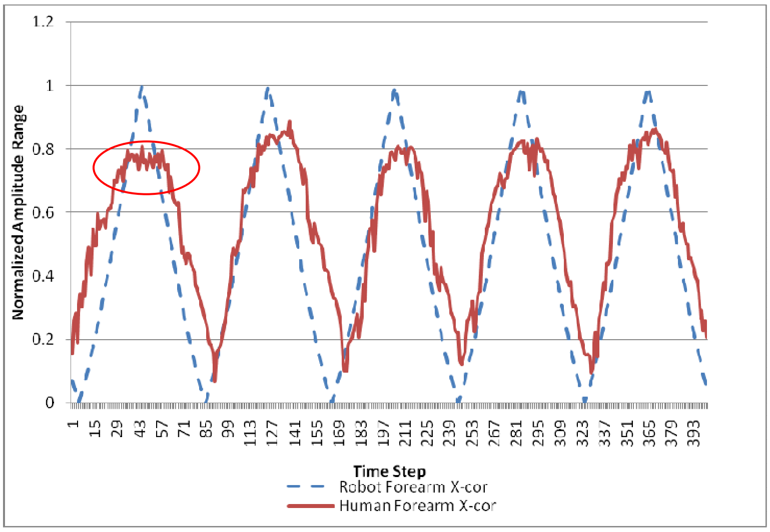

Fig. 3 Robot and Human Forearm X-axis Trajectory

The difference between the *partial*-adaptive binning strategy and the *complete-adaptive binning strategy* is whether the bin size can adapt to the incoming data. The *partial-adaptive binning strategy* has a fixed bin size which only varies as the consequence of the variance of the bin range. The *complete-adaptive binning strategy* allows the bin size to vary in order to ensure that each bin has the same number of data items.

The application of different binning strategies may entirely change the output results from the information distance calculation. As a binning strategy is applied prior to the input of the information distance calculation, changes made to the binning strategy will cause changes to the data distribution features extracted. Hence, the choice of the binning strategy will have an impact on the final output of the entire approach.

2.3 Pre- and Post-binning optimization

This sub-section introduces the processing components of the pre-processing stage excluding the binning strategy component. There are two optional optimization components in this stage. The one prior to the binning strategy component is called *pre-binning-optimization* and the other is called *post-binning-optimization*.

The purpose of *pre-binning-optimization* is to reduce the impact of errors occurring during the data collection stage (such as sensor misdetection). The pre-binning-optimization component consists of two optional sub-components: *curve smoothing* and *normalization*.

Curve smoothing filters the “zig-zag” parts of the human forearm X-axis trajectory curve (illustrated in a ellipse in Figure 3). These “zig-zag” parts may arise from two factors: either the human imitation behavior is not performed smoothly, or the sensors are affected by environmental noise. This may confuse the binning strategy component in detecting the forearm movement tendency. The current strategy applied to curve smoothing is to take the average value of the original data point and its neighbors as the new data point. The effect of this curve smoothing approach is presented in Figure 4.

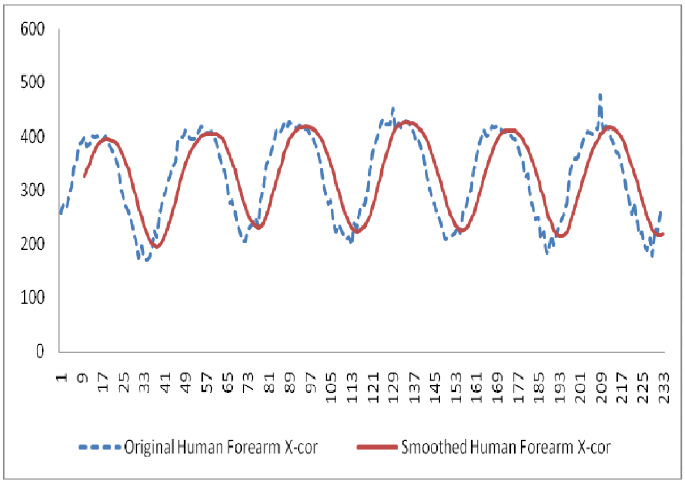

Fig. 4 The Effect of Curve Smoothing

Normalization reduces the impact of inconsistent amplitude, which can be observed in Figure 3. Whether it is appropriate to apply the normalization sub-component depends on the nature of incoming data. If the incoming data curve is supposed to have consistent amplitude, normalization may filter the error in amplitude. Otherwise, application of normalization may cause misleading results. The general strategy is:

1. set the nearest ‘hill’ to 1 and the nearest ‘valley’ to 0;
2. the normalized value of the data between hill and valley = (current data value – original valley data value) / (original hill data value – original valley data value)

The purpose of *post-binning-optimization* is to reduce the data distribution range and therefore enhance the one-to-one relationship between bins from the two data columns being compared. The stronger the one-to-one relationship between two bins is, the more likely they are to be correlated. The higher the correlation of the bins between two data columns, the more likely the two data columns are correlated. That is, in the context of this paper, these two data columns are “similar”.

The current post-binning-optimization methodology we use is called “winner take neighbors”. If bin *a* in column A appears with bin *b* in column B more often than any other bin in column B, then bin *b* will add the number of times its two neighbor bins in column B appear with bin *a* to its own number. Thus, the one-to-one relationship between bin *a* and bin *b* is enhanced.

2.4 Information Distance Calculation

The calculation of information distance between two data columns, usually a pair of corresponding behavior components from the human and robot behavior respectively (for example, the x co-ordinates of the human forearm position and the x co-ordinates of the robot forearm position), is based on the information metric described by Crutchfield [13]. The information distance between two data columns X and Y is defined as the sum of two conditional entropies of these two columns [15]. It can be calculated using the following formula [15]:

$$d(X,Y) = 2 * H(X,Y) - (H(X) + H(Y)) \qquad (1)$$

The entropies presented in the above formula can all be derived from the data distribution features extracted using binning strategies. The joint entropy of column X and Y can be calculated using the frequency distribution matrix and the entropy of X and Y can be calculated from frequency distribution arrays. For more details of the information distance calculation, please refer to [15] and [16].

3 Experimental Setup

The robot used in the following experiments is a minimally expressive humanoid robot called KASPAR, and was developed by the Adaptive Systems Research Group at the University of Hertfordshire. KASPAR is a child-sized humanoid robot with 14 degrees of freedom (8 in head and 6 in arms) [18]. The robot has been designed specifically for the purpose of engaging people in socially interactive behaviour. The robot is e.g. able to perform certain face, head and arm gestures that have been used in human-humanoid imitation games e.g. with children (see figure. 5 and [19]).

Fig. 5: KASPAR (The KASPAR figure is sourced from [18])

A marker-detection toolkit ARToolkit [20] is used in the experiments to detect human body parts. Other object detection approaches such as face detection, color object detection and gray-scale object detection have also been explored. However, the marker-detection approach using ARToolkit is relatively reliable and it can return an object's spatial data to track the position of the object.

As a starting point in the investigation of the method presented, the behavior to be imitated is not expected to be complex. Therefore, the behavior chosen involves only forearm waving while the upper arm is kept stationary. This reduces the complexity of the imitation. The correspondence problem [21] in the imitation behaviors is solved explicitly by mapping human elbow joint angles to robot elbow servo readings.

4 Experiment Results and Analysis

The first set of experiments was conducted to validate the similarity identification method. Please note that in the validation experiments, no optional optimization strategy is applied because all these three experiments are testing the most basic theoretical method.

4.1 Similarity Identification Method Validation Experiments

In order to validate whether this similarity identification model can at least process the data in the right way, a validation process was conducted.

4.1.1 Random Data Validation

The first step of validation is to use randomly generated data columns to check whether the similarity identification model using the partial-adaptive binning (SIM-PB) or complete-adaptive binning (SIM-CB) can identify identical data columns. The results show that both SIM-PB and SIM-CB can find identical data columns as the resulting information distance between them is 0.

4.1.2 Artificial Data Validation

The second step of validation is to use 3-D co-ordinates generated by Matlab [22] which models the waving behaviors between the human and the robot. Compared with the recorded data from the experiments, the modeled data is a much simpler. In this model, the waving behavior of the human and the robot are completely synchronized. There is very little difference between the 3-D position co-ordinates of the human and robot forearm caused by the different arm length settings. The results show that both SIM-PB and SIM-CB can identify very similar behaviors as the resulting information distance between them is 0.

4.1.3 Sine Curve Data Validation

The third step in the validation is to use sine curve data to check how SIM-PB and SIM-CB can handle time step shifts. That is, SIM-PB and SIM-CB will calculate the information distance between the original sine curve and the shifted sine curve. The time step shifts are used to simulate behavioral delay problems in real life. If SIM-PB and SIM-CB can successfully identify similar curves with a small number of time step shifts, it is very likely that they can also identify reasonably delayed imitation behaviors. A sine curve was chosen because it is an ideal continuous periodic data model and the repeated waving behavior is also continuous and periodic. In this validation step, the number of time steps shifted will continuously increase until one entire period is shifted. The performance of SIM-PB and SIM-CB is recorded during shifting. An example of shifted sine curve is presented in Figure 6.

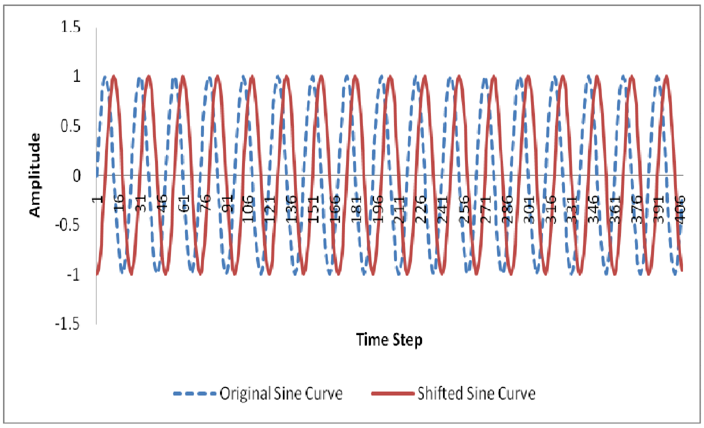

Fig. 6 Sample Sine Curve Used in Sine Curve Data Validation

A result of the validation of SIM-PB is shown in Figure 7. Please note that although there are some special cases due to the assignment of data entries with the same value into the same bin regardless of whether this bin has reached its capacity limit, in general the results outlined are similar to the curve in Figure 7.

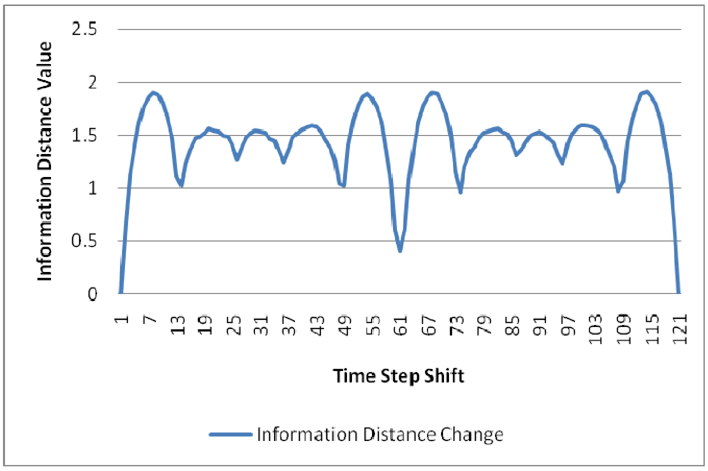

Fig. 7 Sine Curve Data Validation Result

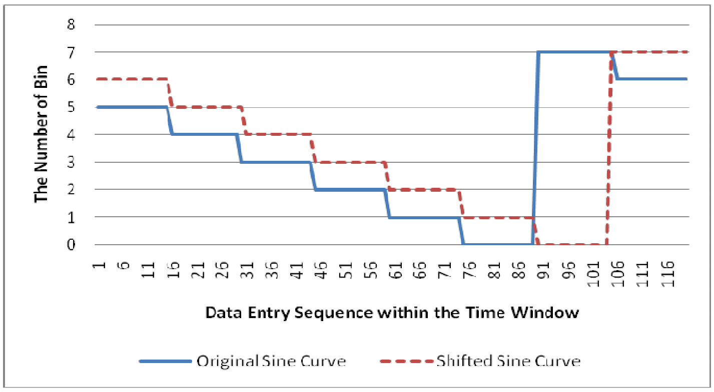

Fig. 8 Relationship between Bins at A Local Minimum

From this figure (which shows the SIM-CB results, SIM-PB gave similar results but is not shown) it is clear that there are three points during the entire process where the information distance between the two curves falls to a low value. As one entire period of the sine curve has 120 time steps, at the 1st time step and 121th time step, the two sine curves are actually on top of each other. That is why the information distance between them is 0. At the 61th time step, when the two sine curves are completely out of phase (become perfectly asynchronous), the information distance between them also goes down. As both mappings (completely in phase and out of phase) indicate the existence of information correlation, the validation can be considered as successful. Thus the method serves to indicate both when the human is (mirror) matching the actions of the robot, and also when the human is matching but is perfectly out of phase, both of which may be considered to be synchronous behaviors. In addition, it also shows that the method is sensitive to the delay because once there is a small number of time step shifts, the information distance rises immediately (and effectively means that the human and the robot are not synchronized). The local minimums in the curve indicate the existence of strong one-to-one relationship. An example is shown in Figure 8. Bin 0, 1, 2, 3 are the descending bins in Figure 8 and bin 7, 6, 5, 4 are the corresponding rising bins.

4.2 Experiments Using Imitation Game Data

The above validation demonstrated that the performance of SIM-PB and SIM-CB met the requirements, i.e. they can successfully identify very similar or identical data columns. Therefore, this similarity identification model was then applied to real human-robot interaction data.

The data used for these experiments were the recordings of three imitation game scenarios. In the first scenario, the human experimenter imitated the forearm waving behavior of the robot (called synchronous imitation). In the second scenario, the human experimenter was imitating the forearm waving behavior of the robot, however, in a different direction (called out of phase imitation – or perfectly asynchronous behavior). In the third scenario, the human experimenter does not do anything when the robot is moving and waves when the robot is doing nothing (called unsynchronized behavior). The results achieved are shown in Figure 9.

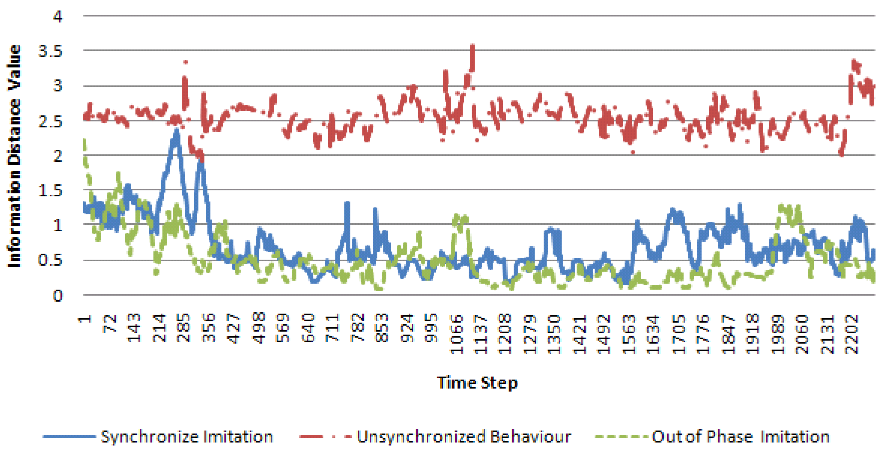

Fig. 9 Result of Experiments Using Imitation Game Data

The results shown in Figure 9 imply the similarity identification model can successfully identify the similarity between robot and human imitation behavior as both the synchronous and out-of-phase imitation curves are visibly separated from the unsynchronized behavior curve. There are two noticeable phenomena: 1) the unsynchronized behavior information distance curve is significantly higher than the synchronized imitation curve and the out-of-phase imitation curve; 2) the synchronized imitation curve is close to the out-of-phase imitation curve. The first phenomenon matches the result expected from information distance calculation: events having less similarity have higher information distance values and vice versa. The second phenomenon matches the results in Figure 7: when two curves are closer to synchronized or completely out of phase, the information distance between them is lower.

The positive results in the experiments also suggest the importance of the binning strategies. If improper binning strategies are used in this model then the results may be very different. The results presented in Figure 10 are derived from the same similarity identification model except for a change of the binning strategy component, in this case the strategy lacks the tendency separation feature. This weakens the one-to-one correspondence between bins and therefore leads to a different result with less clear separations between the curves.

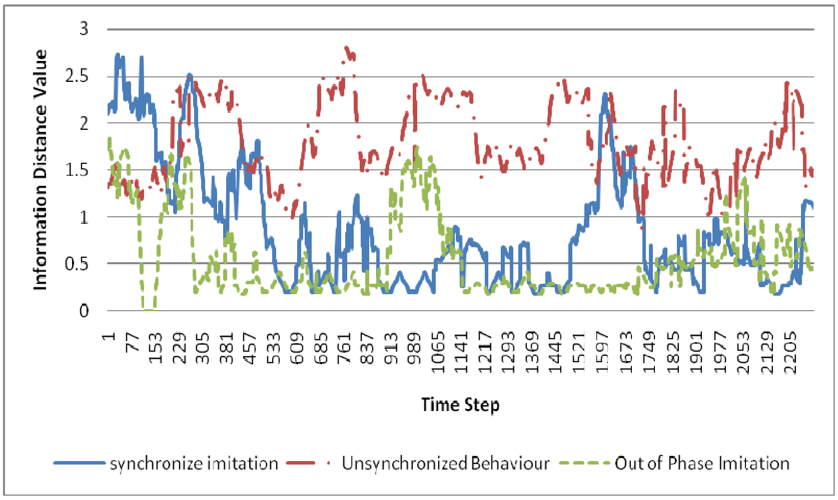

Fig. 10 Results of Using an Improper binning strategy

5 Discussion and Further Work

The experimental results illustrated in section 4 indicate that using the method is able to correctly identify similarity and synchronous behavior between a human and a robot. In real-world human robot imitation interaction, an information distance threshold can be set to explicitly identify the similar and synchronous behaviors.

Therefore, a robot can recognize that a human's action matches its own behavior if the current information distance is within the threshold limit. A mechanism of adapting the threshold is required because different experimental parameter settings and different binning strategies may change the range of information distance.

People may argue what the proper binning strategy is for a particular experiment. Based on this study, a proper binning strategy should retain the most important correlation among the experimental data. Understanding the nature of the experimental data can help to choose or design a proper binning strategy. A validation process then needs to be applied to evaluate the results.

Other approaches, such as Pearson's correlation coefficient calculation, can also identify similar behaviors. However, in this paper, we are not attempting to compare methods, rather we are suggesting this method to complement other approaches. Additionally, we also find that the application of appropriate binning strategies is the key factor that drives the effectiveness of this method. It is because the binning strategy in this information distance method acts as an information filter. An inappropriate binning strategy can cause undesired information loss. Another critical issue of the binning strategy application, which is not presented in this paper, is the choice of the number of bins, where it can be argued as to the number of bins needed, there being no ideal number for all tasks.

Building on the information distance method, further research will investigate how a robot can identify the existence and quality of imitation behaviors during human-robot interaction. Having achieved the above stage, e.g. imitation games that replicate human-infant experiments on the "like me" problem will be conducted to investigate how a robot can acquire and develop social behavior through imitation interaction with humans.

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