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Cognition, Understanding and Behavior

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RobotCub

Development of a Cognitive Humanoid Cub

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D 3.3 A Review on the Generation of Rhythmic and Discrete Movements

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Introduction

Abstract.

Our representation of the world is built upon our sensation and our actions, two processes which are closely related and co-evolve. Studying movement generation is a way to access this representation and to build a model of it. In this review, we aim a describing functionally the generation of movement and its interrelation with somatosensory information in animals, with a special focus on humans. Doing so, we hope to get further insights on the representation of actions in humans.

We focus particularly on the generation of discrete and rhythmic movements, as different mathematical tools are needed to model them. We mainly question the correspondence between discrete and rhythmic movements on one side and voluntary movement and rhythmic automatisms n the other side. Basing ourselves on the concept of generalized motor programs, we conclude by arguing for a classification of movements relative to the parameters needed to characterize them, and, in particular, to timing.

In a second part, we present some interesting concepts for mathematical modeling, and mainly dynamical systems theory and optimal control. In parallel, we recall some of the major models present in the literature, emphasizing their pros and cons and their similarities with what has been observed in humans.

Our aim is to develop a model of movement generation in humans and to apply it to the baby robot iCub, in the framework of the European project RobotCub. We will focus particularly on adaptive, visually-guided locomotion, that we see as discrete corrections, driven by sensory information, of a basic rhythmic locomotor pattern. We strongly believe that taking inspiration from the human system may help us in developing a highly capable robotic controller, this robot being in turn an interesting platform for neuroscientists to test hypotheses on movement generation.

In the framework of the European project RobotCub, we aim at building a functional model of the human motor system for applications in neuroscience and in robotics. Indeed, applying a model of the human motor systems to robots is interesting for at least two reasons:

- Testing hypotheses on movement generation: neuroscience theories relative to movement generation can be corroborated or discarded by testing them on robots;

- Bringing new ideas to control robots: human are capable of dealing with high redundancies and of generating complex movements in a very efficient way that no robot can challenge yet.

In order to achieve this modeling, we start by reviewing some concepts developed by neurobiologists on movement generation and then compare them to the mathematical models present in the literature. Doing so, we hope to contribute a little in bridging the semantic gap between neuroscience and mathematics.
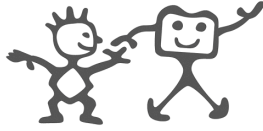
We focus mainly here on the mathematical distinction between discrete (aperiodic and finite) and rhythmic (periodic) movements, a distinction that we think can be explained relatively to their relationship with time. Indeed, discrete movements are defined relatively to their overall duration, with no other specific temporal structure, whereas rhythmic movements are not finite (and thus have not a real duration) and are defined relatively to a temporal repetition of some spatial features.

This distinction seems, at first sight, to find echo in the notions of rhythmic automatisms and voluntary movements defined by neurobiologists. Indeed, three types of movement are distinguished relatively to the way there are produced Kandel et al. (2000):

1. **Reflexes**;

2. **Rhythmic automatisms**[1];

3. **Voluntary movements**.

Reflexes and rhythmic automatisms are spontaneous, whereas voluntary movements are the result of a (motor) plan. Thus, higher cortical areas are required for voluntary movements. As we will see in the following, automatisms are mainly generated at the spinal cord and the brain stem levels whereas the generation of voluntary movement involves areas of the cerebral cortex.

It is interesting to question if rhythmic and discrete movements as kinematic outcomes of the human motor system correspond to, respectively, rhythmic automatisms and voluntary movements as movement generation processes. Indeed typical examples of rhythmic automatisms are breathing, chewing and steady-state locomotion, whereas reaching is one of the key example of a simple voluntary movement.

How far this correspondence between rhythmic movements and rhythmic automatisms in one hand and discrete and voluntary movements in the other holds is a fundamental question. Indeed, Grillner (1985) pointed out that "*whether a motor act should be regarded as voluntary or automatic is not primarily a matter of which type of movements is involved, but rather the context in which it occurs*".

This work is composed of two main parts, namely the biological principles and the mathematical modelings. In the first part, relative to biology, we focus on rhythmic automatisms and voluntary movements and two related concepts, central pattern generators and generalized motor programs. In the second part, mathematical models are considered. Some leading approaches, as dynamical systems theory and optimal control are presented and illustrated by examples from the literature.

Before starting this review, we give a brief overview of the current opinions on the movements generation in humans as found in neurobiology textbooks. We based ourselves on the reference books "Principles of neural science" (4th edition) by Kandel et al. (2000) and "Motor control and learning: a behavioral emphasis" (4th edition) by Schmidt and Lee (2005). Note that this presentation is not meant to be exhaustive, but to give a general idea of the mechanisms involved in movement generation.

Motor structures

Movement generation is achieved through three motor structures organized hierarchically and corresponding to different levels of abstraction. Namely, those structures are:

1. the **cerebral cortex**, which is responsible for defining the motor task;

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[1]Rhythmic automatisms (as motor generation processes) are often referred to as rhythmic movements in the literature. However, to avoid confusion with rhythmic movements as kinematic outcomes of the motor system, we will rather use the term automatisms.
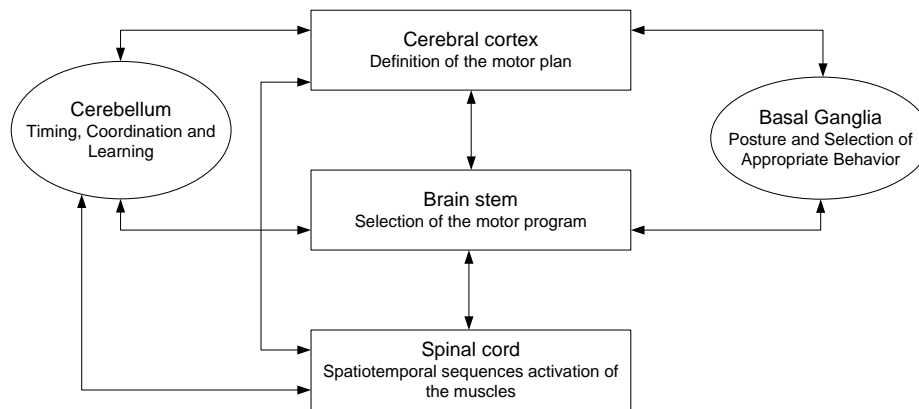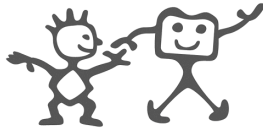
Figure 1: Hierarchical organization of the three motor structures. At the highest level, the cerebral cortex projects directly to the spinal cord and the brain stem. The spinal cord also receives input from the brain stem. Two independent entities, the cerebellum and the basal ganglia have be proven necessary for smooth movements and postures. They interact with both the cerebral cortex and the brain stem.

2. the **brain stem**, which elaborates the motor plan to execute the motor task;

3. the **spinal cord**, which generates the spatiotemporal sequence of muscles activation to execute the task.

In addition, the cerebral cortex and the brain stem are influenced by the **cerebellum** and the **basal ganglia** which can be considered as feedback circuits. All these structures are modulated by somatosensory information, at different levels of complexity, i.e. different levels of processing. However, experiments of deafferented patients have shown that somatosensory feedback is not needed to perform most of the movements (which does not mean that it is not involved in the movement generation).
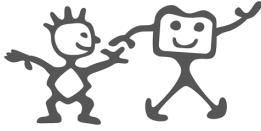
Marsden et al. (1984) have introduced the term **motor program**, which is define as "*a set of muscle commands which are structured before a movement begins and which can be sent to the muscle with the correct timing so that the entire sequence is carried out in the absence of peripheral feedback.*" Motor programs are also used to explain the rapidity with which we react to stimuli and the stereotypy present in human movements. Indeed motor programs may be defined as sequences of simpler, predefined movements, the **motor primitives**, having highly stereotyped spatial and temporal characteristics. Assuming the existence of primitives has the great advantage of reducing the dimensionality of the movement generation problem.

Figure 1 represent a schematic view of the motor structures of the central nervous system (CNS). Note that the cerebellum and the basal ganglia act on the cerebral cortex through the thalamus, which is not represented on the figure for clarity reasons.

Hierarchically, at the lowest level, the spinal cord contains neuronal circuits (the central pattern generators) responsible for reflexes and rhythmic automatisms such as breathing and scratching. Note that those circuits can be modulated by higher areas.

Then, in the middle, the brain stem receives input from the cerebral cortex and project into the spinal cord. It contributes to the control of posture by integrating visual, vestibular, and somatosensory information. It can also control more distal limb muscles involved in goal directed movements.

The cerebral cortex oversees the brain stem and the spinal cord; the primary motor cortex and some other premotor areas project directly to the spinal cord (corticospinal tract) and regulate motor tract from the brain stem.

Observations on patients with lesions in the cerebellum and basal ganglia have shown that the cerebellum is involved in timing and coordination of movements, as well as for learning of new motor programs, whereas basal ganglia is believed to be involved in the motivation and selection of appropriate behavioral responses.

The **somatosensory information** is crucial for movements execution as it provides the representation of the space in which the task has to be performed and also of the state and posture of the body. A constant update of this information is needed to ensure smooth movements.

The somatosensory information is also used to control the movement which is executed. This control can be feedback or feed-forward. **Feedback control** is used by the spinal cord to maintain a given position and to modulate the force needed to perform the task. A **feed-forward control** is used for anticipation and is thus also based on experience; it can also modify the feedback response.

Invariants of motion

A striking features in movement generation is the invariants that have been uncovered by neuroscientists during the last decades and which may reflect the features encoded by motor programs. We recall briefly those invariants here (see Gibet et al. (2004) for details), as any sustainable model of the motor system should reproduce them:

⋄ Invariance of the velocity profile for reaching movement: i.e the global shape of the velocity profile is approximatively bell-shaped with an asymmetry which depends on the speed of movement.

⋄ Isochrony Principle and Fitts' Law: this law reflects the invariance of the execution duration of a movement in relation to its amplitude. In other words, our reaching movements tend to have a duration given by:

$$T = a + b \log_2 \left(\frac{2A}{W} \right) \quad \text{or} \quad T = a + b \log_2 \left(\frac{A}{W} + c \right)$$

with A the amplitude of the movement, W the width of the target, a, b constants determined in an empirical way and $c = 1$ or $c = 1/2$ depending on the author.

⋄ Two-Third Power Law: There is a constant relationship between the kinematics of elliptical motion and the geometrical properties of the trajectory (shown by Viviani and Terzuolo). This leads to the so-called Two-Third Power law:

$$\omega(t) = kC(t)^{2/3}$$

where ω is the angular velocity, C the curvature of the end effector trajectory and k is a constant.

⋄ Smoothness of the movement: the MPG system produce the trajectory which smoothness is maximal by minimizing the mean square of the jerk (Minimum Jerk Model).

Chapter 1

Rhythmic and discrete movements

Mathematically, defining rhythmic and discrete movements is an easy task. Rhythmic refers to periodic signals, discrete to aperiodic ones. However, when considering movements that we actually perform, the task start being tricky, the major problem being that movements are finite in time and that the formal, mathematical definition of periodicity is thus unusable. Moreover inner variability of movements and modulations by the environment (contacts for instance) change the basic nature of the performed trajectory.

In a recent paper, Hogan and Sternad (2007) have developed a taxonomy to classify discrete and rhythmic movements. A discrete movement is defined as a movement which occurs between two postures, where postures stand for a non zero interval of time where (almost) no movement occur. Rhythmic movements are categorized in four subsets, namely periodic, almost periodic, transiently periodic and recurrent. Interested readers are referred to the article for a more precise definition of those terms.
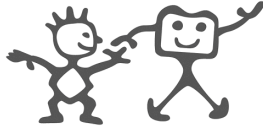
However, as the authors point out in the article, those two definitions are not exclusive. So-called rhythmic movements can occur in between postures, and discrete movements can be repeated so to become periodic. Thus, they propose a measure of smoothness of the trajectory to distinguish discrete from rhythmic movements.

This measure leads them to the conclusion that some rhythmic movements are not discrete (i.e. are not repetition of discrete movements), a conclusion which is in agreement with the result obtained by Schaal et al. (2004) using a human functional neuroimaging experiment. Indeed, some planning cortical areas activated during point-to-point movements of the wrist are not activated when the same joint executes a rhythmic movement.

Even if the measure proposed by Hogan and Sternad (2007) seems to allow to describe the relative "rhythmicity" or "discreteness" of movements, we would like to distinguish rhythmic and discrete movements in a more functional way. Indeed, even if smoothness is a suitable criterion to distinguish the kinematic outcomes, the interesting question from a control theory point of view is where this smoothness comes from and how it can be explained relatively to the structure of the motor system.

Moreover this smoothness quantification speaks more for a continuum between rhythmic and discrete movements than for a distinction. However, if this is the case, it is important for us to define where this continuity lies relatively to the functional process of trajectory generation.

A review of the literature leads to the observation that rhythmic and discrete movements have mainly been studied separately, although some interesting (relatively recent) articles on their combinations exist. This distinction is mainly due, from our point view, to the two following, interlinked factors:

- Rhythmic and discrete movements are representative of two different levels of movement generation, i.e. automatic and voluntary levels. Indeed, locomotion is a typical example of a rhythmic behavior that can be found in species with very low cognitive abilities, whereas a behavior as reaching can only be observed in more evolved ones. This implies different investigation techniques; most of the studies on rhythmic movements have been made on (decerebrates) vertebrates and invertebrates, whereas discrete movement is usually studied using kinematic data or brain imaging recordings of humans or primates.

- Rhythmic and discrete movements have not been studied per se in general, but mainly as outcomes of some specific processes in the trajectory generation, namely the generation of periodic outputs by the central pattern generators (CPGs) at the spinal level for the rhythmic movements and the trajectory encoding process by the brain and the integration of sensory information for discrete movements.

Those two issues make a review of rhythmic and discrete movements difficult in the sense that any comparison between the numerous studies on the subject is almost impossible due to the fact that the methods, the point of view and the neurophysiological level of investigation are different.

It is interesting to question whether the distinction between discrete and rhythmic movement is an artefact due to different scientific approaches or if both movements are indeed produced independently. In this latter case, another question that needs to be answered is whether those two kinematic outcomes may correspond to the two inner processes of automatisms and voluntary movements.

In the next section, we briefly review some important results relative to rhythmic and discrete movements. We start by considering both types of movements separately, or more precisely, we start by considering rhythmic automatisms and voluntary movements, for the reasons mentioned above. We conclude by reporting some interesting results on their combination.

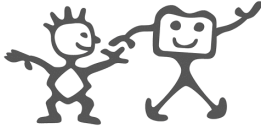## 1.1 Rhythmic movements and automatisms

There is strong evidence that the general pattern of the rhythmic automatisms is mainly produced by the spinal cord in vertebrates (see Stein et al. (1997) for a review). Historically, the assumption that rhythmic motor patterns are generated by the central nervous system has been made by Brown almost one century ago. Since then, experiments with spinalized cats[1] provided further evidences that the main features of rhythmic movements as locomotion are produced by the spinal cord (Stein and Smith (2001); Marder (2000)).

1.1.1 Central pattern generators

Central pattern generators (CPGs)[2] are neural networks capable of producing coordinated patterns of rhythmic activity without any rhythmic inputs from sensory feedback or from higher control centers.

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[1]The term "spinalized cats" refers to cats with their spinal cord transected so to isolate the segments controlling the hind limb musculature from the rest of the CNS. In decerebrate cats the spinal cord and the lower brain stem are isolated from the cerebral cortex. Those two preparations are used to investigate the role of the spinal cord and of the brain stem and cerebellum respectively. To study the role of sensory input together with the sensory and reflex pathways, deafferented and especially immobilized preparations are used. Deafferented means that the sensory innervation of the muscles is prevented by transected the appropriate roots (and thus the appropriate axons). In immobilized preparation muscles is paralyzed using an specific inhibitor, thus impeding any movements. Movements obtain using immobilized animals is often refer to as fictive locomotion.

[2]The whole subsection is taken from Ijspeert (2007).

As reviewed in Delcomyn (1980), they underly many rhythmic behaviors both in invertebrate and vertebrate animals. At the beginning of last century, two different explanations were proposed for the creation of the rhythms underlying locomotion. One explanation was that rhythms are the result of a chain of reflexes in which sensory feedback plays an important role in triggering switches between different parts of a locomotor cycle. The other explanation was that rhythms were generated centrally, i.e. by neural networks that do not require input from the periphery (e.g. sensory neurons) for generating cyclic behavior. Graham Brown for instance proposed a conceptual model called the *half-center* model in which two populations of neurons that are mutually coupled with inhibitory connections and that possess a fatigue mechanism produce alternating rhythmic activity.

There is now very clear evidence that rhythms are generated centrally without requiring sensory information. For instance, one can extract, and isolate from the body, the spinal cord of the lamprey (a primitive fish), and it will produce patterns of activity, called *fictive locomotion*, that are very similar to intact locomotion when activated by simple electrical or chemical stimulation Cohen and Wallen (1980); Grillner (1985). Similar fictive locomotion has been reported in salamander Delvolvé et al. (1999) and frog embryos Soffe and Roberts (1982). More generally CPGs have now been reported in many other animals, see Stein et al. (1997) for a good review.

Similar experiments have also shown that CPGs are distributed networks made of multiple coupled oscillatory centers. Lamprey spinal cords have approx 100 segments, and small sections of the spinal cord (e.g. 1-2 segments) are capable of producing rhythmic activity. The same has been observed in salamanders Delvolvé et al. (1999). This is in agreement with Grillner's proposition that CPGs are organized as coupled unit-burst element with at least one unit per articulation (i.e. per degree of freedom) in the body Grillner (1985). Cheng et al. (1998) report experiments where these units can be divided even further with independent oscillatory centers for flexor and extensor muscles.

While sensory feedback is not needed for generating the rhythms, it plays a very important role for shaping the rhythmic patterns. This is fundamental for keeping CPGs and body movements coordinated. Several experiments demonstrate the important influence of sensory feedback on CPG activity. Mechanically moving the tail of the lamprey will for instance induce CPG activity that is frequency-locked with the frequency of the mechanical movements, and this over a quite large frequency range Viana Di Prisco et al. (1990); McClellan and Jang (1993). Similarly, a mechanically driven treadmill can induce a normally looking walking gait in a decerebrated cat Rossignol (2000), and even induce gait transitions to trot and gallop when the treadmill is accelerated (unpublished work by Graham Brown, as described in Amstrong (1988)). These experiments show a tight coupling between CPG and sensory feedback. This coupling is also visible in the fact that many reflexes are phase-dependent Pearson and Gordon (2000); Pearson (1995); Rossignol et al. (2006) (i.e. reflexes have different effects depending on the timing within a locomotor cycle). This is due to the fact that CPGs and reflex pathways often share interneurons Pearson (1995). See Rossignol et al. (2006) for an in depth review of the interaction of CPGs and sensory feedback mechanisms.

Sensory feedback is also involved in the mechanisms underlying the short-term and long-term adaptation of CPGs according to Pearson (2000). He postulates that the long-term phenomenon are driven by the body and limbs proprioceptors together with central commands and neuromodulators. Kawato (1996) proposed that persistent errors detected by proprioceptors are used to recalibrate the magnitude of the feed forward command.

1.1.2 Locomotion in cats

We now take a closer look at the locomotion of the cat, as this subject has been extensively studied for more than forty years. Experiments on locomotion of neonatally decorticated cats (with the basal
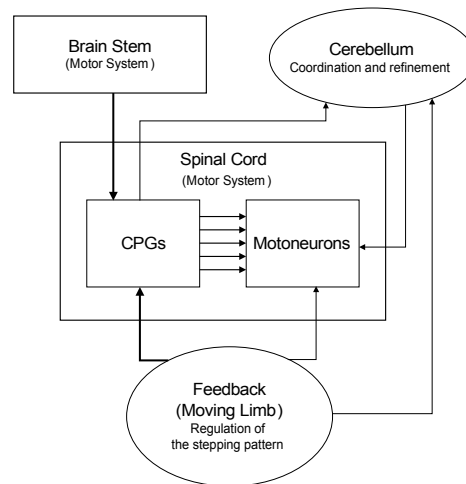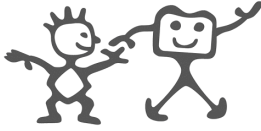
Figure 1.1: Schema of the cat (automatic) locomotion system (without the motor cortex). Thick lines represent the main influence. Note that the cerebellum is essential to obtain refinement of the trajectory and coordination of the limbs. A copy of the signals produced by the CPGs is sent to the cerebellum.

ganglia intact) have shown that they are not only able to perform natural looking locomotion, but also that this locomotion is goal-directed: searching food and eating, taking care of their newborns for female cats, ... However, cats with lesions at the diencephalon (i.e. the part of the brain containing the thalamus and the hypothalamus) loose this ability and tend to move in a machine-like fashion. Basing ourselves on Grillner (1985) we draw a schema of the functional mechanism, based on CPGs, of locomotion generation (see fig 1.1).

Sensory information is involved in the switching from extension to flexion. Indeed, a signal is sent when the limb attains its most posterior position. Although CPGs do not need this signal to start the swing, it allows the cat to modulate the timing and the amplitude of the steps relatively to the environment.

The cerebellum is also an important functional entity in the locomotion system, as cats lacking this part of the brain move in a coarse fashion and encounter equilibrium problems. The cerebellum receives information from the moving limbs and an efference copy (blueprint) from the CPGs. After having processed this information, the cerebellum modulates the motoneurons to adapt the command to the current state of the limb.
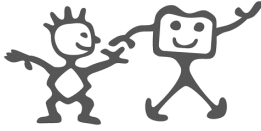
As for the brain stem, it is responsible for the onset and and the termination of the motor behavior, as well as for the modulation of direction and speed.

Finally, the motor cortex is responsible of the adaptive control of the locomotion, i.e. it is necessary for precise foot placement and visually guided locomotion for instance, which require processed sensory information.

1.1.3 From vertebrates to humans

However, we shall ask ourselves how far the results we obtained can be extended to humans. The existence of CPGs in the human system is well accepted nowadays, even if identification of such a spinal network has never been possible yet, to the best of our knowledge. For instance humans with spinal lesions are not able to walk spontaneously.

However, strong evidences is provided by studies on infants (Thelen (2000); Yang et al. (1998);

Lamb and Yang (2000)). Indeed human infants show spontaneous stepping immediately after birth. This phenomenon has been observed in anencephalic infants, providing evidence that circuits responsible for this behavior are located at the spinal or/and at the brain stem level.

Taking inspiration from cat locomotion, the time delay for acquiring voluntary locomotion may be due to the time needed for pathways from the brain stem to the spinal cord to mature during infancy. In addition, complex, adapted locomotion (rather than simple alternation of flexions and extensions) may depend on the development of the pathways between the motor cortex, the brain stem and the cerebellum.

Studies of disabled patients have shown that in the absence of sensory information, gross movement control is preserved, even if peripheral information is necessary for precise movement organization and control (for reviews, see Jeannerod (1988) or Gandevia and Burke (1992)). Thus, sensory information seems not to be essential to produce a basic pattern of movement.

1.2 Discrete and voluntary movements

Discrete movements are movements which can be referred to as movements with an identifiable beginning and end. Reaching is a key example of such a behavior and it has been extensively studied in the framework of voluntary movements both in primates and humans.

Voluntary movements have several features that distinguish them from automatisms and reflexes. First of all, center to the notion of voluntary movement, is the will to perform a given behavioral task. Then, not only the movement is performed to achieve the task in the best possible way, but responses to sensory stimuli are also modulated relatively to this task. Finally, they are improved by experience and learning (Kandel et al. (2000)).
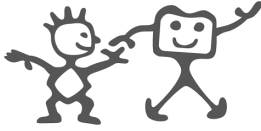
1.2.1 The cerebral motor structure

We discuss here in more details the motor structure in the cerebral cortex. As before, we base ourselves on Kandel et al. (2000). We focus on the functional architecture of this structure, that is depicted on figure 1.2. The motor cortex can be subdivided in two areas, the primary motor cortex and the premotor cortex (M1), this latter being formed of the lateral (dorsal and ventral) premotor areas (PMd and PMv) and of the supplementary motor area (SMA).

It is interesting to note, that the sensory structure, the motor cortex and the SMA are organized in a somatotopic way, i.e. parts of the body are represented by different location in the motor areas and that the size of the area is representative of their degree of cortical control. This map is plastic, in the sense that it evolves with learning.

Some cortical axons connect with the spinal cord, both with motor neurons and interneurons. Signals for premotor cortex and from the primary motor cortex seem to overlap at the spinal level.

Experiments using electric simulations have shown that one of the role of the primary motor cortex is to switch on and off certain groups of muscles (low level control). A certain muscle can be activated by stimulating several locations, and stimulating a particular location activate several muscles. It seems that in the monkey motor cortex, the center of the area activates distal muscles, then a surrounding area allows to control together proximal and distal muscles while the more peripheral area are related to proximal muscles only.

Moreover, the primary motor cortex is involved in the specification of the features of the movements and of their modification relatively to changing conditions (high level control). An interesting
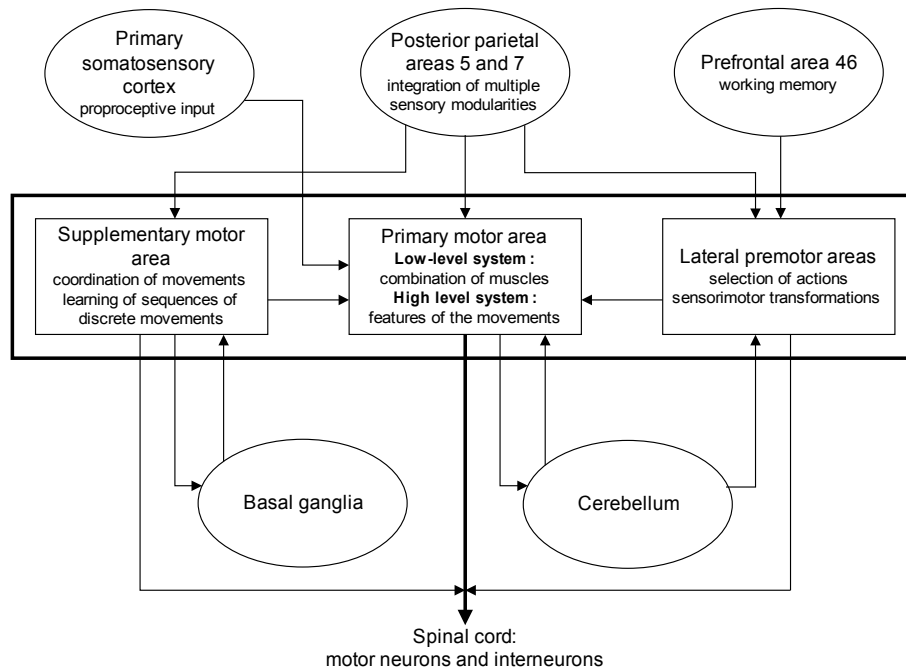
Figure 1.2: Functional schema of the architecture of the cerebral motor structure, represented by the big box in the middle. Main afferent and efferent connections are also shown on the graph.
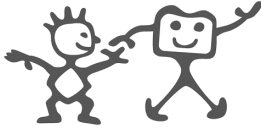
feature is that single neurons seem to encode force, whereas direction is encoded by population of neurons.

As for the premotor cortex, the SMA (and the presupplementary motor areas, not shown on the figure) are involved in learning sequences of movement and in timing. The lateral areas contribute to the processing of the sensorimotor information (together with areas 5 and 7 of the postparietal cortex) and to the selection of action.

The prefrontal area 46 provides working memory, i.e. a short time memory lasting the time of the task, for instance for location of obstacles. The sensorimotor cortex provide then the essential proprioceptive information to the motor cortex.

Note that this "traditional" view on the functioning of motor cortex is challenged by the works of Graziano et al. (2002), who made experiments where regions of the brain of the monkey where stimulated for 500 ms (the time scale of normal reaching and grasping movements), this duration being longer than in traditional studies. They found out that those simulations were resulting in a complex movement ending in the same location whatever the initial position of the limb was. They conclude from this that instead of encoding regions of the body, the motor cortex was a representation of different complex postures.

In addition, recent research on mirror neurons (Fogassi et al. (2005); Nakahara and Miyashita (2005)) have shown that specific but different neurons are activated in the motor cortex when, for instance, food is grasped to be eaten or to be put in a trash bin. One can thus assume that purposes of behaviors are encoded in the motor cortex rather than trajectories. Evidences for a similar functional coding, in the sense of a definition of the movement relatively to its goal, has been uncovered by Bizzi's group at the spinal level (Bizzi et al. (2002)). This lead to the model of force fields and equilibrium point, that we discuss in the next section.

1.2.2 Force fields in the frog and equilibrium point

Bizzi's group provided some evidence for the concept of motor primitives. Indeed, they brought to light that movements were generated in a modular way by the spinal cord in frogs (Tresch et al. (1999); Bizzi et al. (2002)). More precisely, stimulating specific areas of the spinal cord, they observed that the limb was moved in the direction of the same endpoint point (the **equilibrium point**), whatever the initial position of the limb was. They called the set of the vectors corresponding to the directions obtained by the stimulation **force fields**. Furthermore they found out that stimulating two areas simultaneously was almost equivalent to a simple linear combination of the vector of the force fields corresponding to the stimulation.

Thus, small set of modules organized by the spinal cord have been identified; by simply combining those modules, a wider range of movements can be produced. Moreover, experiments have shown that such combinations techniques are used by the spinalized frog during its natural behaviors (Kargo and Giszter (2000)).

However, the most interesting part of the concept of force fields is that movements seems to be encoded relatively to a particular final position, or final goal, rather than relatively to the exact trajectory. This idea is present in the equilibrium point hypothesis (EPH)(see for instance Feldman et al. (1998)). In short, in the EPH, only the endpoint of each muscle is defined and the typical characteristics of the trajectories (e.g the invariants of motion we listed in the introduction) are emerging properties of the dynamics of the body.

The EPH starting point is a modeling of muscles as springs and dampers. Indeed, movements are produced by a contraction of an agonist muscle and the extension of the antagonist, which corresponds, in the spring and damper model, to a change of the respective rest position of the springs representing the muscles, the damping term allowing to prevent oscillations. Movements arise when the shift of the rest positions exceeds a given threshold.

However, in order to forecast movements, the threshold has to be a complex function reflecting the history and the current state of the muscle. Thus models such as the λ model have been introduced (Feldman et al. (1998)). In this model, the actual activation threshold λ^* is governed by the following equation:
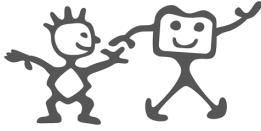
$$\lambda^* = \lambda + \mu v + \rho + f(t)$$

where λ and μ are control commands, ρ represents non central terms due to inter-muscular interactions and $f(t)$ reflects the history dependent change in the threshold.

Note that the EPH is a disputed model and there is still an open debate on this subject. Indeed, on the contrary of the EPH, an alternative school of thought called the internal dynamics approach emphasizes the role of learning, i.e. of motor program progressively acquired through experience and optimizing some given criteria (see for instance Hinder and Milner (2003) and the answer by Feldman and Latash (2005)). In addition, experiments on deafferented patients (Nougier et al. (1996)) seem to provide evidence for amplitude coding rather than position coding.

1.2.3 Generalized motor programs

We now discuss an important concept in movement generation, that is motor equivalence. This term has been introduced by Hebb in early 50s to refer to the observation than whatever the size of your handwriting and the part of the body with which you write (dominant or nondominant hand, mouth), it appears the same (Lashley; Bernstein (1947)). This leads to the conclusion that the brain does not

encode muscles activation or effector trajectory but rather spatial features of it[3].

Another evidence for spatial features encoding is the fact that a reaching movement will always have approximatively the same duration for instance, whatever the distance to the target is. Moreover the velocity and acceleration profile will be the same.

Those observations has lead to the definition of **generalized motor programs** (GMP)(Schmidt and Lee (2005)). Recall that a motor program is a predefined sequence of stereotyped actions called motor primitives. In a generalized motor pattern, the spatial features of the primitives depend on some parameters that have to be specified to the program (i.e. the duration of the movement, the size of the handwriting) .

This generalization is helpful to deal with two limitations of the usual motor programs, namely the size of the storage (how many programs?) and the specification problems (how to adapt?). We have chosen to talk about GMP in the discrete section as the specification of the parameters is a voluntary (though not necessarily conscious) process.

A GMP can be seen as a class of action (Schmidt and Lee (2005)), stored in memory and responsible for a unique pattern of activity, this pattern being formed of the characteristic features of the activity being performed but being subject to parametrization.

The question of knowing which features the GMP encodes is not trivial. A convenient view point is the impulse-timing hypothesis which postulates that GMP controls (indirectly) when to turn on and off muscles and the force of the contraction of those muscles. Under this assumption, the GMP may contain the following fixed structures, according to Schmidt and Lee (2005):

- the sequence of actions (which is independent of the muscles; it is postulated that the same program is used to write with the right hand or the mouth for instance)

- the temporal structure or relative timing

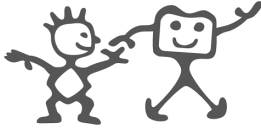- the force structure or relative forces

and the following parameters

- the overall duration

- the overall force

- the muscle-selection parameters.

Note that if the concept of GMPs does not exclude rhythmic movements, in the sense that patterns normally generated automatically can be used in a GMP, however the specification of parameters is voluntary and thus the resulting movement can no longer be considered as a pure automatism.

1.3   Combination of discrete and rhythmic movements

Most studies on the combination of rhythmic and discrete movements are built on the same scheme: a particular joint (the finger or the elbow generally) has to be moved from an initial to a target position (discrete movement) while oscillating (rhythmic movement). The oscillation is either physiological

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[3]At first sight, this conclusion seem to speak for the internal dynamics model rather than for the equilibrium point hypothesis. However, levels of abstraction should be taken in account here, in the sense that a specification of spatial, meaningful features at the cortical level may be represented through equilibrium points specification (i.e via-points) at the spinal level.

(Goodman and Kelso; Adamovich et al. (1994); Michaels and Bongers; Sternad et al. (2000)) or pathological (Wierzbicka et al.;Elble et al. (1994); Staude et al.). For a very nice review on this subject, the reader is referred to Sternad (2007).

In all these experiments, an entrainment effect was observed, that is the discrete movement is phase-coupled with the rhythmic movement, in the sense that the onset of the discrete movement occurs preferably (though not always) during a specific phase window of the oscillations. Goodman and Kelso showed that this phase window correspond to the peak of momentum of the oscillations in the direction of the discrete movement. Interestingly, it is a well known fact that professional pistol shooters press the trigger in phase with their involuntary tremor, while beginners try to immobilize themselves before shooting.

In terms of EMG, the burst initiating the discrete movement occurs approximatively at the time where the EMG activity for the rhythmic movement would have been expected without this perturbation. This effect is thus refer to as "burst synchronization" by De Rugy and Sternad (2003).

As for the effect of the discrete movement on the rhythmic one, Adamovich et al. (1994) observed the following:

1. The oscillations rapidly attenuate during the discrete movement and resume after the peak velocity of the discrete movement;

2. There is a phase resetting of the oscillations after its attenuation;

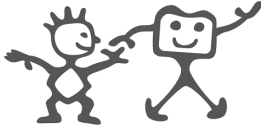3. The frequency tend to be higher after the discrete movement.

Moreover, they found that once the discrete movement is initiated, it is performed independently from the rhythmic one. Basing themselves on monotonic hypothesis (St-Onge et al. (1993)), i.e. an hypothesis according to which the command of the discrete movement stops at the time of its peak velocity, they conclude that discrete and rhythmic movements are excluding each other at the neural level, in the sense that they cannot co-occur. However, their kinematic outcome outlast them and thus overlap.

Sternad et al. (2000)performed the same experiment, although at lower frequencies (2-3Hz instead of 5-7Hz). In their experiment, they observed a significant influence of the rhythmic movement on the discrete movement (lower frequencies of oscillations lead to longer discrete movements), which is in contradiction with the results obtained by Adamovich et al. (1994). Moreover the higher frequency observed in the previous study appeared to be a transient phenomenon. Thus their hypothesis is that both movements may co-occur, but that in this case they are inhibiting each other.

It is difficult to draw a conclusion on whether the movements occurs in sequential or superimposed way at this stage. However, here the main task is the discrete one and it may thus be interesting to consider a situation where the predominant task is the rhythmic one. For instance, in foot placement during locomotion, the discrete correction of the step has to occur in a way so to perturb as little as possible the rhythmic ongoing movement. The results may thus be different and possibly complementary to the ones reported here.

1.4 Concluding remarks on the notion of timing

We have started this chapter by questioning whether rhythmic and discrete movements were two different, independent processes or if they were two outcomes of the same process. In a recent article, Schaal et al. (2004) have provided evidence that rhythmic (arm) movements are not discrete, as some cortical areas involved in discrete movements where not involved in the execution of the rhythmic

ones. However it still may be that discrete movement is a special case of rhythmic movements, i.e. discrete movements may be truncated rhythmic movements.

We have seen that voluntary movements and rhythmic automatisms are known to result from different processes (i.e. from different neural pathways). How far this distinction correspond to discrete and rhythmic movements is a possible way of questioning their differences. Indeed, when considering "simple" movements, answering is easy: rhythmic movements are automatisms and discrete movements are voluntary ones.

However, let us take the simple example of visually-guided locomotion, which interest us particularly in the framework of the RobotCub project (Righetti and Ijspeert (2006); Degallier et al. (2007)). Turning left or right, avoiding obstacles some steps in advance or climbing stairs are situations where the rhythmic movement is modified in a discrete fashion to adapt to cerebral representation of the task. Now, the resulting kinematic outcome may be considered as (transiently) rhythmic or as discrete (because it is not periodic in the strict sense).
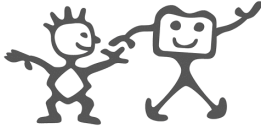
We have seen that Hogan and Sternad (2007) have proposed a measure of smoothness to classify discrete and rhythmic movements. A more functional alternative may be to use the notion of timing. Indeed, recent studies have postulated that different timing processes could be involved depending on the features of the task to be performed. Named automatic or cognitively controlled timing (Lewis and Miall (2003)), or emergent and explicit timing (Spencer et al. (2005)), the two processes are involved in the timing of respectively periodic, highly predictable movements at the spinal level and of discontinuous and unpredictable ones at the cerebral one.

Indeed, patients suffering from a lesion at the cerebellum exhibit a timing impairment which is observable only in the case of discontinuous movements, in the sense of non-repetitive movement (Spencer et al. (2005)). In addition, in their study using neuroimaging of the brain, Schaal et al. (2004) observed a much higher activity in the cerebellum (among other areas) for discrete movements than for rhythmic ones, even when performed by the same joint, in the same range.

Thus the notion of timing may offer a functional criterion to characterize the level of rhythmicity or discreteness of a movements, or we would rather say to the level of automatism or volition involved for a given kinematic outcome. Indeed in the case of adaptive locomotion, climbing stairs for instance, the timing of the discrete correction of the rhythmic one is explicit (it is defined relatively to the visual perception of the stairs for instance) whereas the timing for the automatic part of the locomotion is controlled at the spinal level.

Some other concepts, as the level of processing of the needed information may be relevant. Indeed, Schneider and Shiffrin (1977) distinguish the controlled and the automatic processing of information; the controlled process is relatively slow, volitional and attention demanding (as in obstacle avoidance, for instance, where the visual information has to be processed), whereas the automatic one is fast, not volitional (and often unavoidable (Underwood and Everatt (1996))) and demands no attention (as the contact information of a foot with the ground which is fed back into the CPGs in locomotion). This difference in information processing seems to correspond to the different types of movements: no processing of the information for reflexes, an automatic processing for rhythmic automatisms and finally a controlled processing for voluntary movements. It may thus also be used to classify the movements, for instance accordingly to the brain areas involved in the elaboration of the motor plan (or in the parametrization of the GMP).

Note that under the modularity hypothesis, and assuming that motor primitives are superimposed (as postulated by Sternad's group), one can argue that a kinematic outcome is composed of discrete and rhythmic parts and thus that the question of knowing whether it is more rhythmic or more discrete is less relevant. This hypothesis is moreover consistent with the notion of GMP and leads to the following, three-layered architecture for a controller

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- a planning level: decision of the motor program to be executed and of the parameters of this program;

- a motor program: the selection of the motor primitives and their relative timing;

- a trajectory generation level: the temporal sequences of the angles (or torques).

To a certain extent, this architecture represents the three structure of the general motor system we describe at the beginning. However, we still need to define the needed motor primitives. In accordance to the fact that rhythmic movements are not special case of discrete movements, two hypotheses are possible:

- two different motor primitives are required for discrete and rhythmic movements or

- discrete movements are truncated rhythmic movements.

In the latter case, timing consideration are of great importance: indeed in rhythmic movement the general duration of the movement has no influence on the timing between the muscles activation, whereas in discrete movement the general duration of the movement modifies this relative timing. Thus discrete and rhythmic movements may still be considered as different processes, but at the timing level rather than at the generation one.

To conclude, in this section we have tried to get further insight into the generation of rhythmic and discrete movements. We believe that while rhythmic movements are probably grounded in automatisms and discrete movements are mostly learned and thus originally voluntary, this distinction is not clear anymore in everyday movements. Indeed, outside the laboratory, the fact of considering movements such as locomotion as automatisms is obviously questionable, as well as considering reaching (and grasping) as voluntary.

This observation reflects the interrelation between the different motor structures; each of them correspond to a different level of representation of the task, which modulate each others. Movements initially learned become automatic with time and automatisms modulate the learning of new tasks.

As for classification of discrete and rhythmic movements, we have suggested to characterize them relatively to the parameters needed to define it, and in particular timing. For us, this characterization seems coherent with the notion of motor programs encoding key features of the movements.

Chapter 2

Mathematical models for the generation of discrete and rhythmic movements

In this chapter, we present the some of the existing models of the combination of discrete and rhythmic movements. Almost all of them are built using dynamical system theory (Schöner and Santos (2001); Schaal et al. (2000); De Rugy and Sternad (2003) and Ijspeert et al. (2002)), except the approach from Biess et al. (2006), which is based on optimal control, and the model by Staude et al. to explain the phase entertainment effect.

We thus start with a brief tutorial on dynamical systems theory to make the understanding of those approaches easier. We then present the different models based on this theory. Finally, after a short introduction to optimal control, we present the work of Biess et al. (2006). We conclude by some considerations on the possibilities of combining both approaches.

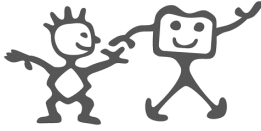## 2.1 The dynamical systems approach

We present here, in brief, some concepts of dynamical systems theory. Indeed, this approach has proven to be well suited for the control of robot in many applications (see, for instance, Schöner and Dose (1992); Tani et al. (2004); Fukuoka et al. (2003); Ijspeert et al. (2003); Pongas et al. (2005); Degallier et al. (2006); Righetti and Ijspeert (2006)), moreover, as we will see, dynamical systems are particularly well-suited for the modeling of discrete and rhythmic movements, as they have two characteristic types of solutions, namely, fixed point and limit cycles, which correspond respectively to discrete and rhythmic signals.

In addition, dynamical systems have several interesting properties: (i) they generate trajectories in real-time with little computation[1], (ii) they can smoothly modulate the trajectories when control parameters (e.g. parameters defining the goal posture) are changed, and (iii) they can be designed to have useful stability properties against perturbations.

2.1.1 Short tutorial on dynamical systems

This short tutorial is meant to introduce some key features of dynamical systems that will prove to be useful for modeling purposes. It is thus by far not exhaustive. It is based on the excellent introductory textbook on dynamical systems of Strogatz (2001).

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[1]Optimization techniques, for instance, lead to excellent results but the computation time needed to find a solution is often such that those techniques are not suitable for real-time control.

By dynamical systems, it is meant either differential equations (continuous time) or iterative maps (discrete time). Here we focus on the continuous case for obvious reasons. Moreover, as we are concerned with the evolution of the system with time (and not with other variables) we are only interested in ordinary differential equations[2], i.e. in systems where all the differentiations are made relatively to the same variable, time in our case. A typical example is the equation of a damped harmonic oscillator,

$$m\ddot{x}_1 + b\dot{x}_1 + kx_1 = 0,$$

where $\dot{x}_1$ stands for $\frac{dx_1(t)}{dt}$ and $\ddot{x}_1$ for $\frac{d^2x_1(t)}{dt^2}$. Note that here the function x_1 depends only on time (t) so that the derivatives are not partial. The solution $x_1(t)$ of the system is obtained either by analytical or by numerical integration, i.e. using iterative methods as Euler or Runge-Kutta. In any case, the initial conditions are needed, i.e. $x_1(0)$ and $\dot{x}_1(0)$ have to be specified.

The order of the equation is the order of the highest derivative appearing in the equation. The equation mentioned above is of second order for instance.

On nonlinearity

The equation of the damped harmonic oscillator we mentioned above is said to be linear, because it can be turned into a system of linear equations by setting $\dot{x}_1 = x_2$, this leading to the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{b}{m}x_2 - \frac{k}{m}x_1 \end{aligned}$$

where no nonlinear terms, as x_2^2, x_1x_2 or $\sin(x_1)$ for instance, appear. Note that this process turn an equation of order n into n first order equations.

An example of a nonlinear system is the equation of a swinging pendulum of length L

$$\ddot{x}_1 + \frac{g}{L}sin(x_1) = 0.$$

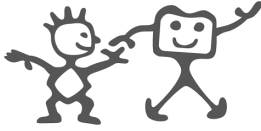Using the same trick as before, i.e. setting $x_2 = \dot{x}_1$ we obtain

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}sin(x_1) \end{aligned}$$

which is obviously not linear, because of the sine term. Even this simple equation is difficult to solve analytically. Indeed, contrarily to linear systems, nonlinear systems can not be decomposed relatively to each equation because of their nonlinear interaction. However this difficulty is exactly what makes them appropriate to model natural phenomena, where everything is interlinked.

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[2]In opposition with partial differential equations, which are equations involving partial derivation of functions of several variables. A typical example is the wave equation derived by d'Alembert

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial u^2} = 0,$$

where c is a constant corresponding to the propagation speed of the wave.

Types of solutions

Dynamical systems have to major types of stable solutions: **fixed points** and and **limit cycles**. Those types of solution are a nice representation of discrete and rhythmic movements respectively. We will thus discuss in more details those two types of solutions that we illustrate with two well-known examples of systems exhibiting such solutions, namely **Matsuoka oscillators** and the **VITE model**. We have chosen those two systems in particular because they are inspired from biological observations and are widely used in the literature.

Fixed-point systems

A system of differential equations $\dot{\vec{x}} = \vec{f}(\vec{x})$ is said to have a fixed point solution $\vec{x}^*$ if $\vec{f}(\vec{x^*}) = 0$. Thereafter, arrows on vectors are omitted.

Different types of stability for fixed points x^* are usually distinguished, namely

1. **locally attracting** fixed points, that is points to which any trajectory starting sufficiently close will eventually converge, i.e. x^* is attracting if $\exists \delta > 0$ such that if $||x(0) - x^*|| < \delta$, then $\lim_{x \to \infty} x(t) = x^*$;

2. **globally attracting** fixed points, that is points to which any trajectory will eventually converge, i.e. x^* is attracting if $\forall x(0)$, $\lim_{x \to \infty} x(t) = x^*$;

3. **Liapunov stable** fixed points, that is points such than any trajectory starting sufficiently close to them will remain close to them, i.e. x^* is Liapunov stable if $\forall \varepsilon > 0$, $\exists \delta > 0$ such that if $||x(0) - x^*|| < \delta$, then $||x(t) - x^*|| < \varepsilon, \forall t \geq 0$.

4. **asymptotically stable** fixed points, which are points that are both Liapunov stable and attracting.

Note that a solution which is Liapunov stable but not attracting is sometimes called **neutrally stable**.

To illustrate the notion of fixed point, we present in the following the VITE model, which is a bio-inspired model as we will see. This model is frequently used to generate discrete movements in the literature.

> **The VITE Model:**
> **A Neural Command Circuit for Generating Arm and Articulator Trajectories**
> Daniel Bullock and Stephen Grossberg,
> in *Dynamic Patterns in Complex Systems*, 1988.

The VITE (**V**ector **I**ntegration **T**o **E**ndpoint) model has been developed by Bullock and Grossberg to simulate planned and passive arm movements and has later been extended to speech articulator movements. In this model, the final length of the muscles is specified (the Target Position Command (TPC)) - and thus this model is closed to the Equilibrium Point Hypothesis - together with the overall speed of the movement (the GO command). These quantities are converted into trajectories through automatic processes modeled as the following set of differential equations for each of the degrees of freedom (dofs) involved in the generation of the movement:

$$\begin{cases} \dot{v} = \alpha(T - p - v) \\ \dot{p} = g[v]^+ \end{cases}$$
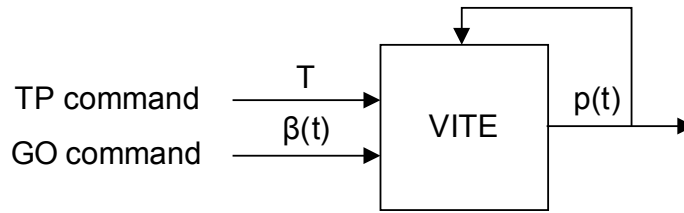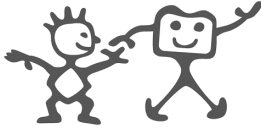
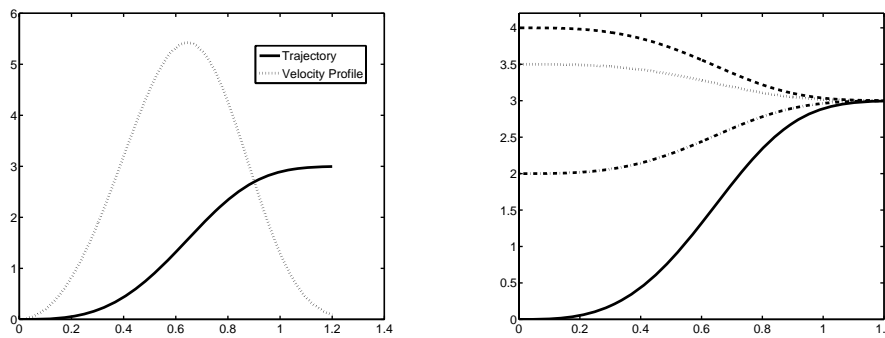Figure 2.1: Schema of the architecture of the VITE model

Figure 2.2: **Left**: A typical trajectory obtained using the VITE model (position is plotted relatively to time). Here $\alpha = 100$ and $\gamma = 0.5$. The trajectory asymptotically converges to the target $T = 3$. The bell-shaped velocity profile is also shown on the graph. **Right**: Illustration of the notion of an asymptotically stable fixed point. Whatever the initial condition is ($p(0) = 0$, 2, 3.5 or 4 here), the trajectory eventually converges to the target $T = 3$, in the same amount of time.

where $[v]^+ = \max(0, v)$[3], α is a constant and T is the target of the movement (given by the TPC).

p represents the current position of the dof (and thus $p(t)$ represents the trajectory of the dof) and gv is the rate of change of this position (and thus $g(t)v(t)$ is the speed of the movement).

This system has a global, asymptotically stable fixed point in $p = T$. The variable v is used to update the present position p with respect to the remaining distance to the target $(T - p)$ so that it converges to T at a rate controlled by $g(t)$. $g(t)$ is a time-varying function corresponding to the GO command, as controlling the rate of convergence allows for synchronization of the multiple muscles involved in a given movement, even if the amount of contraction differs from one to another.

Figure 2.2 illustrates the output of such a system. More precisely we have considered the equivalent system[4]:

$$\begin{cases} \dot{v} = \alpha(\alpha/4(T - p) - v) \\ \dot{p} = (e^{\gamma t} - 1)^2 v \end{cases}$$

where the GO command $g(t) = (e^{\gamma t} - 1)^2$ is chosen so to ensure that the velocity profile is bell-shaped.

Thus this systems allows for a very easy synchronized control of the all dofs which trajectory is ensure to eventually converge to the specified target. However, as the duration of the movement is constant whatever the displacement is, attention must be paid to the velocity which is not bounded in

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[3]As v represents the activity of hypothetical neurons controlling the difference between the target position and the present position (the difference vector), it can only be positive or null.

[4]The restriction to the positive part of v is omitted for simplicity reason.
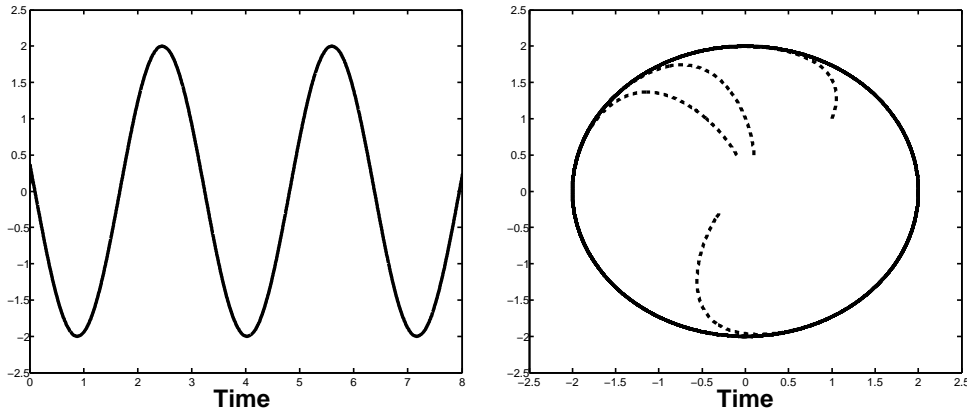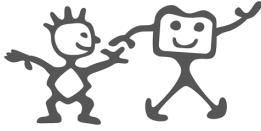
Figure 2.3: **Left:** The output trajectory of an Hopf oscillator is a sine. **Right:** When plotting y against x, the limit cycle appears clearly. The space (x, y) is called the phase plan.

this system.

Limit cycle systems

The second type of solutions we are particularly interested in are limit cycle, i.e. asymptotically stable periodic solutions. Systems exhibiting such solutions are often called oscillators. For a more detailed description of oscillators, the reader is referred to Buchli et al. (2006).

To illustrate this type of solution and the notion of couplings and synchronization, we start by considering an Hopf oscillator, which is governed by the following set of equations

$$\begin{cases} \dot{x} = (\mu - r^2)x + \omega y \\ \dot{y} = (\mu - r^2)y - \omega x \end{cases}$$

The limit cycle of such an oscillator for $\mu > 0$ is a sine of frequency ω and of amplitude $\sqrt{\mu}$, as illustrated on figure 2.3. For $\mu < 0$, the system has a stable fixed point in $x = 0$[5]. This system can be easily coupled to another oscillator with variables $\bar{x}$ and $\bar{y}$[6], in the following way:

$$\begin{cases} \dot{x} = (\mu - r^2)x + \omega y + c(\cos(\theta)\bar{x} - \sin(\theta)\bar{y}) \\ \dot{y} = (\mu - r^2)y - \omega x + c(\sin(\theta)\bar{x} + \cos(\theta)\bar{y}) \end{cases}$$

where the c is the gain of the coupling and θ is the phase offset between both signals. Illustration of this phenomenon are given on the left of figure 2.4.

Under some stability constraints, such couplings will induce a stable phase-locked behavior between the two oscillators, i.e. a synchronized behavior, which is a nice property to have in many applications, where one needs coordinated movements (e.g. in locomotion). **Synchronization** is a very important feature of coupled dynamical systems and itis interesting to take advantage of it when designing models for coordinated movement controllers. We refer the reader to the book of Pikovsky et al. (2001) for basic introduction to synchronization concepts.

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[5]This qualitative change of the solution (i.e. between a limit cycle and a fixed point here), is called a bifurcation, and more precisely an Hopf bifurcation in this case.

[6]To avoid high modifications of the amplitude of the movement due to the couplings, the vector $(\bar{x}, \bar{y})$ is often normalized.
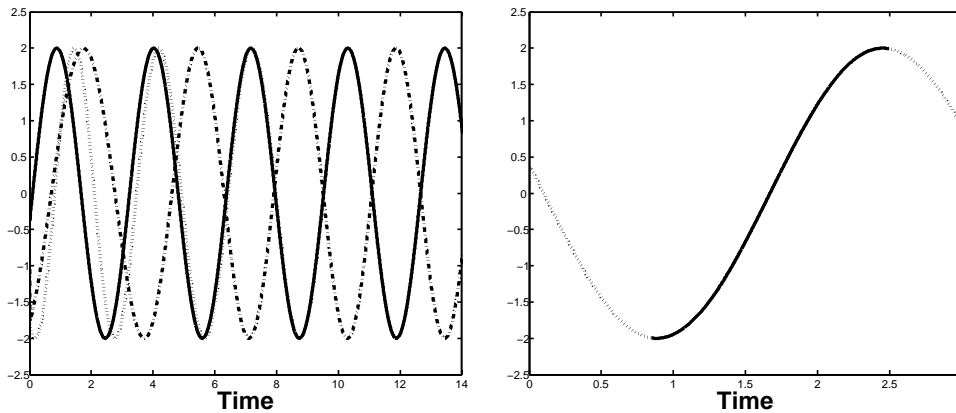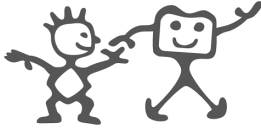
Figure 2.4: **Left:** Two oscillators with the same initial conditions (in dash-dot line and in dot line) are coupled with a reference oscillator (plain line) with a phase offset of π and 0 respectively. The synchronization occurs after an initial transient phase. **Right:** Discrete movements can be modeled by truncated rhythmic movements.

Recent advances in dynamical system theory are worth mentioning in this context since they provide tools for the design of synchronized coupled dynamical systems. It is worth noticing here the fundamental work of Golubitsky and Stewart on the generic existence of symmetric periodic solutions in networks of coupled dynamical systems. The interest of the approach is that by looking at the structure of the coupling between each dynamical system and more specifically by looking at the symmetries of the system, they are able to prove the existence of periodic solutions having the desired phase locked relations, in a manner that is independent of the internal dynamics of the each dynamical system (Golubitsky et al. (1999);Golubitsky and Stewart (2002);Golubitsky et al. (2005)). A complementary approach from Pham and Slotine (2007) also introduces simple sufficient conditions for asymptotically stable synchronization using nonlinear contraction theory.

Note that is is also possible to generate a discrete movement by taking a well chosen part of the periodic signal, see 2.4 on the right. This may be a way of modulating both types of movement through the same generator if the hypothesis that discrete movements are truncated rhythmic movements is made. We will present such a model, by Schöner and Santos (2001), later in this section.

Sustained Oscillations Generated by Mutually Inhibiting Neurons with Adaptation

Kiyotoshi Matsuoka,
in *Biol. Cybern*, 1985.

In this article, Matsuoka presents a oscillator modeling mutually inhibiting neurons. This model is widely used in the literature and is referred to as the Matsuoka oscillator. It is based on the concepts of membrane potential and firing rate. It also takes in account the phenomenon of adaptation, also referred as fatigue or self-inhibition, which account for the gradual decrease of the output of a firing neuron after the initial, rapid increase of the signal.

We present here the model for two mutually inhibiting and self-inhibiting neurons, which can be used to represent two neurons controlling two antagonist muscles, a model which is depicted of figure 2.5. The set of equations modeling the activity of a neuron inhibited by an other neuron, which output is denoted by $\bar{x}$, are the following:

$$\begin{cases} \tau_1 \dot{x} = -x - \beta y + S - \omega[\bar{x}]^+ \\ \tau_2 \dot{y} = -y + [x]^+ \end{cases}$$
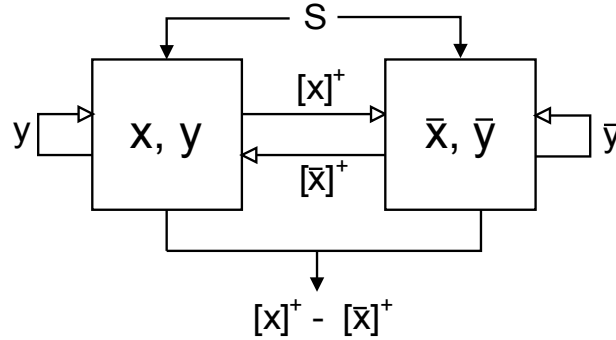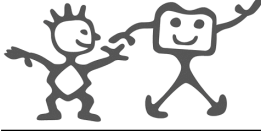
Figure 2.5: Schema of the Matsuoka's model of two inhibiting neurons with self-inhibition. Empty arrows denotes inhibitive couplings.
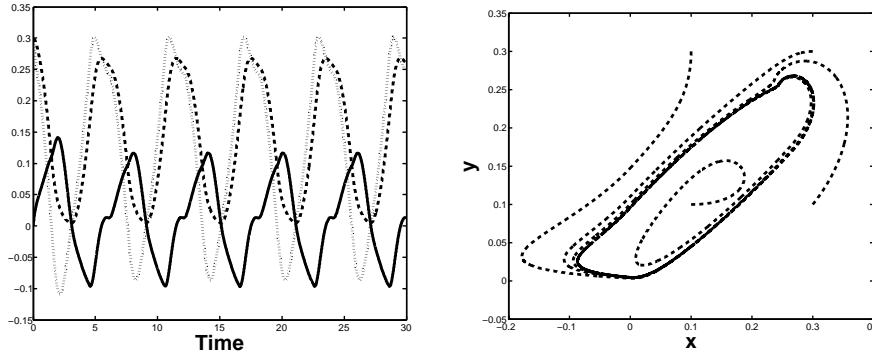

Figure 2.6: **Left:** Trajectories generated by a couple of inhibiting and self-inhibiting Matsuoka oscillators. In plain the difference between the two oscillators is represented, whereas in dot and in dash are the outputs of both oscillators. **Right:** Illustration of the limit cycle of a neuron in the Matsuoka oscillator. Whatever the initial condition is ($(x_0, y_0) \in \{(0.1, 0.1), (0.1, 0.3), (0.3, 0.1), (0.3, 0.3)\}$ here), the trajectory eventually converges to the limit cycle (in plain).

where the dynamics of the other neuron are governed by the same set of equations, i.e.

$$\begin{cases} \tau_1 \dot{\bar{x}} = -\bar{x} - \beta \bar{y} + S - \omega [x]^+ \\ \tau_2 \dot{\bar{y}} = -\bar{y} + [\bar{x}]^+ \end{cases}$$
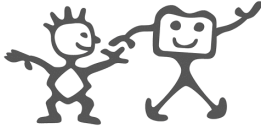
where $[x]^+$ and $[\bar{x}]^+$ are the output of both neurons (i.e. the firing rate of the neurons), x and $\bar{x}$ represent the membrane potential[7] and y ($\bar{y}$) controls the self-inhibition of the neuron. S is the tonic input, β and ω are the coupling weights of the self-inhibition and of the inhibition by the other neuron respectively and τ_1 and τ_2 are time constants. The general output χ of the system is given by $\chi = [x]^+ - [\bar{x}]^+$.

This system is oscillatory under the following conditions (Arsenio (2000))

$$\beta > \omega - 1, \qquad \omega > 1 + \frac{\tau_1}{\tau_2}.$$

and its amplitude depends linearly on the tonic input S and its frequency is proportional to $\frac{1}{\sqrt{\tau_1 \tau_2}}$. The resulting limit cycle can be shaped using parameters τ_1, τ_2, S, β and ω. An illustration of such a

---

[7]Note that neurons have a threshold value (θ) below which they do not fire, thus the variable x has to be understood has the difference between the actual membrane potential (x_{mp}) and the threshold, i.e. $x = x_{mp} - \theta$.

limit cycle and the corresponding trajectories are given in figure 2.6, where we coupled to inhibiting oscillators, which are given by the set of equations:

$$\begin{cases} \dot{x} = -x - 3y + S - 3.5[\bar{x}]^+ \\ 0.5\dot{y} = -y + [x]^+ \end{cases}$$

and

$$\begin{cases} \dot{\bar{x}} = -\bar{x} - 3\bar{y} + S - 3.5[x]^+ \\ 0.5\dot{\bar{y}} = -\bar{y} + [\bar{x}]^+ \end{cases}$$

and the general output is given by $\chi = [x]^+ - [\bar{x}]^+$.

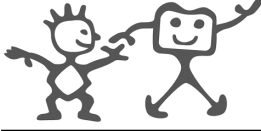**Some considerations on central pattern generators**

As was said before, CPGs are organized as a network of coupled oscillatory centers. The dynamics of this network is mainly embedded in the topology of the network, i.e. in the links between the centers, rather than in the local mechanisms underlying the rhythm generation. This feature is present in the dynamical system theory and most of the CPGs models used in robotics involve networks of coupled differential equations. Such models include connectionist models, vector maps and systems of coupled oscillators (see Ijspeert (2007) for details).

CPGs for modeling locomotion have been widely used in the literature. For a nice example, we refer the reader to the model developed by Righetti and Ijspeert (2006) for baby crawling.

The main disadvantage of the approach is that dynamical systems are difficult to design, due to the fact that the system itself does not encode only one trajectory, but a whole space of trajectories corresponding to the different possible states of the system. Indeed, there is no clear methodology on how to design a system of differential equations whose solution is a given, arbitrary trajectory. Some researches in this direction have been taken on and have lead to promising results, see for instance Golubitsky et al. (2005), Ermentrout and Kopell (1994)), Slotine et al. (2004) and Righetti et al. (2006).

However, under the assumption that the movement is generated in a modular way, design difficulties is not a drawback, in the sense that, in this case, the complexity of the trajectory emerges from the combination of the modules rather than from the modules themselves, which encodes simple, stereotypical trajectories. Moreover, this facilitates the required stability analysis of the dynamical systems.

We now present some interesting approaches on dynamical systems through some chosen articles. It is aimed to underline some interesting properties of such modelings rather than to be a complete summary of the articles.

D 3.3 A Review on the Generation of
Rhythmic and Discrete Movements
Development of a Cognitive Humanoid Cub

2.1.2   Modeling discrete and rhythmic motor primitives

We present here some existing models for the generation of rhythmic and discrete movements. In the first model, discrete movement can be seen as a special case of rhythmic movement, whereas in the second one, two separate motor primitives exist.

> **Control of movement time and sequential action through attractor dynamics:**
> **A simulation study demonstrating object interception and coordination**
>
> Gregor Schöner and Cristina Santos,
> in the proc. of the *9th Intelligent Symposium on Intelligent Robotic Systems*, 2001.

We present here the model developed by Schöner and Santos. This model is built to generate discrete movements, but is based on limit cycles, which makes it easily extendable for the generation of rhythmic movements.

In this model, discrete and rhythmic movements are both modeled using limit cycles, i.e. discrete movements are interrupted rhythmic movements, which implies that timing is crucial. Thus they develop a two-layered system, i.e. a layer able to generate both oscillations and stationary states ("timing layer") and another layer controlling the switching between those two states ("neural dynamics control"). Note that the timing of the movement has to be very precise in this model, which may be a drawback when implemented on robots.
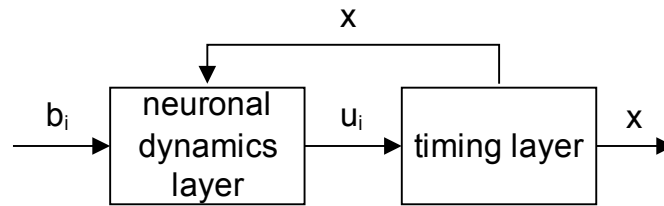

Figure 2.7: Schema of the control architecture of the model of Schöner and Santos.

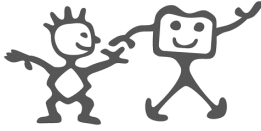Mathematically, the equations of the timing layer are given by:

$$\begin{cases} \dot{x} = -a|u_{\text{init}}|(x - x_{\text{init}}) + |u_{\text{hopf}}|(b(1 - r^2)x - \omega y) - a|u_{\text{final}}|(x - x_{\text{final}}) + \text{gwn} \\ \dot{y} = a|u_{\text{init}}|y + |u_{\text{hopf}}|(b(1 - r^2)y - \omega x) - a|u_{\text{final}}|y + \text{gwn} \end{cases}$$

where a and b are constant controlling the speed of convergence of the system and "gwn" stands for gaussian white noise. In this system, $|u_i|$ (i=initial, hopf, final) represents neurons which are never active (i.e. equal to one) at the same time. The timing of activation of the three "neurons" is controlled by the neuronal dynamics which are given by the following equations:

$$\begin{cases} \alpha \dot{u}_{\text{init}} = \mu_{\text{init}} u_{\text{init}} - |\mu_{\text{init}}| u_{\text{init}}^3 - c(u_{\text{hopf}}^2 + u_{\text{final}}^2)u_{\text{init}} + \text{gwn} \\ \alpha \dot{u}_{\text{hopf}} = \mu_{\text{hopf}} u_{\text{hopf}} - |\mu_{\text{hopf}}| u_{\text{hopf}}^3 - c(u_{\text{init}}^2 + u_{\text{final}}^2)u_{\text{hopf}} + \text{gwn} \\ \alpha \dot{u}_{\text{final}} = \mu_{\text{final}} u_{\text{final}} - |\mu_{\text{final}}| u_{\text{final}}^3 - c(u_{\text{init}}^2 + u_{\text{hopf}}^2)u_{\text{final}} + \text{gwn} \end{cases}$$

Each equation correspond to the normal form of a degenerate pitchfork bifurcation controlled by parameters μ_i[8] with an extra term to ensure that only one neuron is active, i.e any solution with more than one neuron active is destabilized.

---

[8]That is the system has one stable solution ($u = 0$) when μ_i is negative and two stable ones ($u_i = 1$ and $u_i = -1$) when μ_i is positive.

The neuronal dynamics allow to decompose the movement in three phases by only changing the parameters μ_i (i=initial, hopf, final):

- Initiation phase (μ_{init} is dominant); the right part of the equation correspond to a stable stationary solution at $x = x_{\text{init}}$, i.e. the system stays in its initial position (initial postural state).

- Movement phase (μ_{hopf} is dominant); the right part of the equation corresponds to an Hopf oscillator of amplitude equals to 1 and frequency equals to ω. The referential of the system is chosen so that $x_{\text{init}} = 0$ and $x_{\text{final}} = 1$, so that the amplitude of the limit cycle corresponds to the length of the movement. Timing is done such that the system follows approximatively half of the limit cycle and thus reaches the target.

- Termination phase (μ_{final} is dominant); the right part of the equation correspond to a stable stationary solution at $x = x_{\text{final}}$, i.e. the system reaches and stabilizes at its target position (target postural state).

The switching between the different phases is achieved using the parameter μ_i; the neuron with the larger μ_i will take the advantage on the others. We can rewrite μ_i under the form $\mu_i = m + n\beta_i$, where $\beta_i \in [0, 1]$ and m, n are constant depending on the system. With this notation, in the absence of movement, $b_{\text{init}} = 1$ and $b_{\text{hopf}} = 0 = b_{\text{final}}$. Initiation of the movement is triggered by setting $b_{\text{init}} = 0$ and $b_{\text{hopf}} = \sigma(x - x_{\text{cri}})$ where σ is a sigmoid function which tends to 0 when x tends to $x_{\text{cri}} = 0.7x_{\text{final}}(= 0.7)$. Finally, $b_{\text{final}} = 1 - b_{\text{hopf}}$.

Note that α is set too a much higher values than a, b so that the two systems have two different time scales. In this way, x can be considered as a parameter at the neuronal level, whereas transient states of the neurons (u_i) can be neglected at the timing level.

We think that the two-layered architecture of this system is interesting because it induces dynamics for both the generation of the trajectories and the specification of the parameters of those trajectories. This approach is consistent with the different levels of movement generation in humans, where the cerebral cortex provides the relevant parameters to the brain stem and the spinal cord which generates the joint trajectories accordingly. Moreover, the mutual influence of those dynamics reflect the interrelation between the different entities of the human motor system.
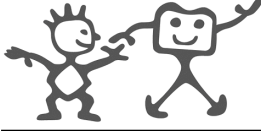

Nonlinear dynamical systems as movement primitives

Stefan Schaal, Shinya Kotosaka and Dagmar Sternad,
in the proc. of the *IEEE International conference on Humanoid Robotics*, 2000

Schaal, Kotosaka and Sternad have developed a model based on a programmable pattern generators (PPGs), which is an extension of the concept of motor pattern generators (or generalized motor programs). Indeed, PPGs are seen as generators of trajectories with some predefined characteristics but also with some open, task-specific parameters that need to be specified.

In this model, movements are generated through three layers, as illustrated on figure 2.8: the command signal which is triggered by the difference between the actual position and the desired target position or amplitude for discrete or rhythmic movement respectively. Then, a generator for discrete movements and a couple of inhibiting Matsuoka oscillators for rhythmic movements are used to generate a desired velocity signal. Finally, this velocity is integrated to obtain a desired displacement (in terms of position).

In this model the movement is triggered by the distance the dof has to cover (through the difference vector), this distance being used to determine the speed of movement (a larger distance involving a

larger speed). This modeling is consistent with the observation that reaching movements have always the same duration, whatever the distance is.
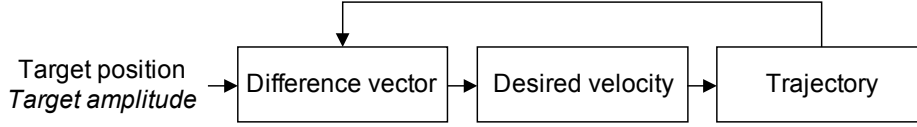

Figure 2.8: Schema of the three steps architecture of the model developed by Schaal et al. The architecture, although not the equations, are the same for discrete and rhythmic movements.

In addition, the discrete system is modeled by a fixed-point system, where the target value is called the rest position θ_0. A point to point movement is thus seen has a modification of the rest position of the arm, a approach which is close to the EPH. Note that the new rest position can be specified either directly in the joint space, or firstly in the cartesian space and then in the joint space through inverse kinematics transformations. This latter approach is more convenient as most discrete task are defined relatively to the cartesian space rather than relatively to the joint space.

The fixed-point system for one dof is given by the following set of (weakly) nonlinear differential equations

$$
\begin{cases}
\Delta v_i = [T_i - \theta_{o,i}]^+ \\
\dot{v}_i = a_v(-v_i + \Delta v_i) \\
\dot{x}_i = -a_x x_i + (v_i - x_i)c_o \\
\dot{y}_i = -a_y y_i + (x_i - y_i)c_o \\
\dot{r}_i = a_r(-r_i + (1 - r_i)bv_i) \\
\dot{z}_i = -a_z z_i + (y_i - z_i)(1 - r_i)c_o \\
\dot{\theta}_{o,i} = a_p([z_i]^+ - [z_j]^+)c_o
\end{cases}
$$

where $i \in \{1, 2\}$ and $j \in \{2, 1\}$ and are representative of the agonist and antagonist muscles and of their reciprocal influence. In the two first equations (similar to the VITE model), a difference vector Δv_i, which represents the difference between the target and the actual position, is passed through a differential equation to simulate the activation pattern of the muscles which depends on this difference vector. Then, the second couple of equations smoothes the signal, with y_i representing the velocity of the signal. The variable c_0 can be tuned to modify the speed of the movements. As the velocity y_i is (strongly) asymmetric in this model (whereas it is approximately bell-shaped in reality) a variable r is added to correct this asymmetry. Finally z_i is the desired, unscaled velocity signal that is finally turned into a angle displacement by integration of the last equation.

As for the rhythmic movement, it is modeled as oscillations around the rest position defined by the discrete system. The limit cycle system used is given by the following set of equations:

$$
\begin{cases}
\Delta \omega_i = [A - \theta_{r,j}]^+ \\
\dot{\xi}_i = a_\xi(-\xi_i + \Delta \omega_i) \\
\dot{\psi}_i = -a_\psi \psi_i + (\xi_i + \psi_i + \beta \zeta_i + w[\psi_j]^+ + K_i^\alpha)c_r \\
\dot{\zeta}_i = -\frac{a_\psi}{5}\zeta_i + ([\psi_i]^+ - \zeta_i)\frac{c_r}{5} \\
\dot{\theta}_{r,i} = \psi_i \\
\theta_r = c_r([\theta_{r,1}]^+ - [\theta_{r,2}]^+)
\end{cases}
$$

In this system, the two first equations are equivalent to the difference vector of the first equation, except that this time it correspond to the difference between the desired amplitude and the actual position. Then, a couple of inhibitory Matsuoka oscillators are used to generate a velocity signal
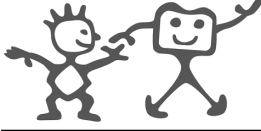
(whereas in the original model, it was used to generate a position signal). In this model, the two oscillators correspond to two antagonist muscles. Finally, the difference between the two oscillators is integrated to obtain the desired trajectory.

To control precisely the offset between each oscillator, they are all coupled to a reference oscillator α of parameters θ_1^α and θ_2^α. The coupling terms K_i^α ($i \in \{1, 2\}$) of the previous system are then given by:

$$
\begin{aligned}
K_1^\alpha &= -A^\alpha(w_{11}^\alpha[\theta_{r,1}^\alpha]^+ + w_{21}^\alpha[\theta_2^\alpha]^+) \\
K_2^\alpha &= -A^\alpha(w_{12}^\alpha[\theta_{r,1}^\alpha]^+ + w_{22}^\alpha[\theta_2^\alpha]^+)
\end{aligned}
$$

where the weight matrix $W^\alpha(\gamma)$ is given by

$$
W^\alpha(\gamma) = \left(\begin{array}{cc} \gamma & 1 - \gamma \\ 1 - \gamma & \gamma \end{array} \right)
$$

with $\gamma \in [0, 1]$. Setting γ to 0 will result in in-phase oscillations and setting it to 1 in an offset of π. Intermediate values of γ correspond to intermediate offsets. A^α controls the coupling weight to the reference oscillator.
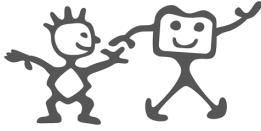
This model achieves to reproduce movements containing many human-like features, as coordination of movements or the bell-shaped velocity profile for instance. Its particularly interesting because it provides a robust mathematical way of modeling the concept of generalized motor programs and the concept of motor primitives.

We have chosen to present here those two approaches because they do not only take advantages of the numerous nice properties of dynamical systems, but they also provide a nice modeling of some important features in human movement generation. Indeed, the model developed by Schöner and Santos interlinks the dynamics of the movement itself and those of the parameters of the movement, which reflects the interrelation between the different levels of representation of a motor plan observed in humans. As for the model presented by Schaal, Kotosaka and Sternad, it is close to the model of generalized motor programs in biology and provides an promising way of tackling the complexity and the adaptiveness of human movements.

2.1.3 The phase entrainment effect: dynamical systems and classic approaches

In the previous chapter (i.e. in section 1.3), we have mentioned the phase entertainment effect, i.e. the fact that, during a rhythmic movement, the onset of a discrete movement is phase-coupled to the rhythmic movement. More precisely, the EMG burst corresponding to the discrete movement occurs approximatively at the same time as the rhythmic EMG burst would have been expected without this initiation (De Rugy and Sternad (2003)). Kinematically, this onset seems to likely take benefit from the rhythmic movement, i.e. to occur as a continuation of the rhythmic movement, rather than acting against it.

We present here two different models which explain the phase entrainment effect differently. In the model proposed by De Rugy and Sternad (2003), the mechanism underlying the coupling between the onset of the discrete movement and the rhythmic movement is due to synchronization tendencies, whereas in the model proposed by Staude et al. this phase-locked behavior is due to threshold phenomena. This latter model does not use dynamical systems, but is presented here as an alternative approach.

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The discontinuous nature of motor execution II.
Merging discrete and rhythmic movements in a single-joint system
- the phase entrainment effect

Gerhard Staude, Reinhard Dengler and Werner Wolf,
in *Biol. Cybern.*, 2002

</div>

Staude et al.'s have built a model on the assumption that superimposition of discrete and rhythmic movements "*involves a threshold-linear mechanism at the level of motor execution: the dynamic properties of the muscles and its reflex connections may cause nonlinear interactions between simultaneously executed motor task, thus restraining a general linear superimposition of kinematic patterns*". Mathematically, the summation of the command signal for discrete and rhythmic movements is filtered through a limiter function which restrains the output of the sum to positive values smaller than a given upper bound.

More precisely, this model, illustrated on figure 2.10, is built on the assumption that the level of muscle activation Σ^+ is given by a threshold function of the type:

$$\Sigma^+(t) = [\phi(t) + c_d(t) + c_r(t)]_0^\gamma$$

where $c_d(t)$ and $c_r(t)$ represent the control commands for discrete and rhythmic movements respectively, $\phi(t)$ is a positional and velocity-dependant sensory feedback and

$$[x]_0^\gamma = \begin{cases} 0 & \text{if } x \leq 0 \\ \gamma & \text{if } x \geq \gamma \\ x & \text{otherwise} \end{cases}$$
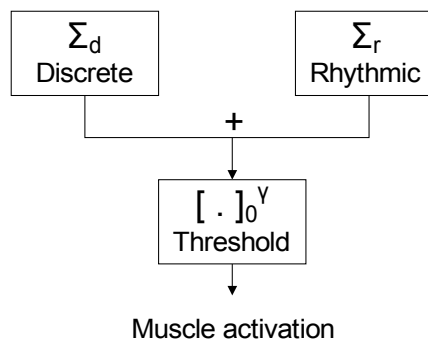
is the limiter function introducing the nonlinearities.

Figure 2.9: Schema of the phase entertainment model developed by Staude et al. The entertainment effect is due to nonlinearities introduced by a threshold function.

For discrete movement, the initiation of the signal is modeled as a rapid shift of the control signal $c_d(t)$, i.e.

$$c_d(t) = \begin{cases} c_{\text{in}}, & \text{if } t < t_{\text{go}} \\ c_{\text{in}} + \omega(t - t_{\text{go}}, & \text{if } t \geq t_{\text{go}} \end{cases}$$

where t_{go} denotes the onset time of the shift of $c_d(t)$, c_{in} is a negative constant and ω is the rate of change corresponding to neural dynamics limitations. As the point of focus here is the initiation of the
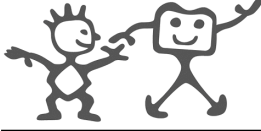
movement, no bound for ωt need to be specified and the feedback can be considered to be constant (i.e. $\phi(t) = \phi_{\text{in}}$) and thus the system can be turned into

$$\Sigma_d(t) = \left\{ \begin{array}{ll} \Sigma_{\text{in}}, & \text{if } t < t_{\text{go}} \\ \Sigma_{\text{in}} + \omega(t - t_{\text{go}}), & \text{if } t \geq t_{\text{go}} \end{array} \right.$$

where $\Sigma_{\text{in}} = c_{\text{in}} + \phi_{\text{in}}$.

A periodic pattern of activation is assumed to be responsible for the generation of rhythmic movement, i.e., as a first approximation,

$$c_r(t) = A_r \cos\left(\frac{2\pi}{T}t + \psi_r\right).$$

For purely rhythmic movements,

$$\Sigma_r(t) = \phi_r(t) + c_r(t)$$

where

$$\phi_r(t) = \beta_p x(t) + \beta_v \dot{x}(t),$$

i.e the feedback is composed of a term proportional to the position $x(t)$ with a gain of β_p and a term proportional to the velocity $\dot{x}_t$ with a gain of β_v. Since $x(t)$ represents a rhythmic activation of the muscle it can be modeled, as an approximation, as a cosine and thus $\dot{x}(t)$ is a sine. All the terms of $\Sigma_r(t)$ are thus sine or cosine of same frequency, and they can thus be rewritten under the form

$$\Sigma_r(t) = A \cos\left(\frac{2\pi}{T}t + \psi_0\right).$$

To simplify analysis, the control commands are normalized to the amplitude A and expressed relatively to the phase of the rhythmic signal rather than relatively to time, i.e. using $\phi = \frac{2\pi}{T}t + \phi_0$, it is obtained:
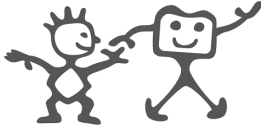
$$\Sigma_r(t) = cos(\psi)$$

and

$$\Sigma_d(t) = \left\{ \begin{array}{ll} k_1, & \text{if } \psi < \zeta \\ k_1 + k_2(\psi - \zeta), & \text{if } \psi \geq \zeta \end{array} \right.$$

where $k_1 = \frac{\Sigma_{\text{in}}}{A}$, $k_2 = \frac{T\omega}{2\pi A}$ and $\zeta = \frac{2\pi}{T}t_{\text{go}} + \phi_0$.

The role of the limiter is thus twofold: firstly, both signals have to be simultaneously increasing to ensure that their sum is positive, leading to a coordination of the movements in terms of direction, and secondly the upper limit bounds the signal due to the discrete movement.

This model not only succeed in reproducing the observed activation patterns in human, but also in forecasting the observed probability of onset relatively to the phase time of the oscillations.

Interaction between discrete and rhythmic movements:
reaction time and phase of discrete movement initiation during oscillatory movements

Aymar de Rugy and Dagmar Sternad,
in *Brain Research*, 2003

In this model, a couple of inhibiting Matsuoka oscillators is used to generate both discrete and rhythmic movements by modifying the tonic input of the system. Thus discrete movements are a special case of rhythmic movement. The architecture of this model is depicted on figure 2.10. The entertainment effect "comes for free" due to synchronization effects between the two Matsuoka oscillators (i.e. the discrete movement occurs as a continuation of the rhythmic movement).
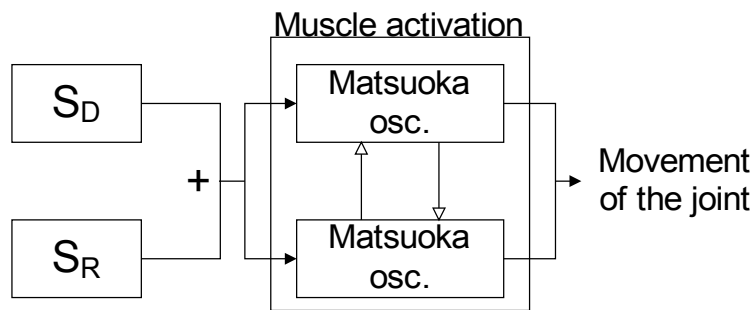

Figure 2.10: Schema of the phase entertainment model developed by De Rugy et al. In this model, the entertainment effect is due to synchronization effects .

Indeed, recall that this oscillator is governed by the following equations (for one neuron):

$$\begin{cases} \tau_1 \dot{x} = -x - \beta y + S - \omega[\bar{x}]^+ \\ \tau_2 \dot{y} = -y + [x]^+ \end{cases}$$

where S is the tonic input, which is fixed for rhythmic movement ($S = S_R$ =cste). The movement of the joint is then derived using the equation

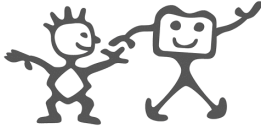$$I\ddot{\theta} + \gamma\dot{\theta} - T = 0$$

where $T = h_T([x]^+ - [\bar{x}]^+)$, I being the moment of inertia and γ a damping term. For discrete movement, the tonic input S_D is modeled as a pulse followed by an exponential decay:

$$\tau_3 \dot{S}_D = -S_D + p_D$$

where p_D is the peak value of the pulse and τ_3 a time constant. This signal results in a damped oscillation which, with well-tuned parameters (i.e. for a critical damping), generates a typical discrete movement. To generate discrete and rhythmic movement simultaneously, the tonic input are simply summed together, i.e $S = S_D + S_R$. The output of this signal is synchronized through the dynamics of the Matsuoka oscillators.

Both approaches achieve to explain successfully the phase entertainment effect. In the model of de Rugy and Sternad, the synchronization is due to the fact that both trajectories are generated through the same (dynamical) system, whereas in the model of Staude et al., both commands are generated separately and the observed synchronization is caused by a nonlinear combination of the signals.

2.1.4 Learning with dynamical systems

Learning and imitation is a very important feature in movement generation. We thus conclude this section on dynamical systems by a model which allow imitation using dynamical systems.

> **Movement Imitation with Nonlinear Dynamical Systems in Human Robots**
> Auke Ijspeert, Jun Nakanishi and Stefan Schaal,
> in the proc. of the *IEEE International Conference on Robotics and Automation*, 2002.

In this article, Ijspeert, Nakanishi and Schaal introduced a approach to imitation learning that combines dynamical systems and local weight regression techniques. We present here the model for discrete movement, but the general idea of the method can be extended to rhythmic movement. Please refer to Ijspeert et al. (2003) for such an approach.

If we consider a reaching movement for instance, the idea in this model is to use a known stable dynamical system to obtain a trajectory that converges to the desired target, and then to modulate the dynamics of this trajectory to obtain a trajectory which is as close as possible to a given, desired trajectory. This latter is achieved by minimizing the difference between the desired and the actual trajectory through a weight regression on a linear combination of Gaussian kernel functions.

More precisley, the discrete motor pattern generator[9] is given by

$$\begin{cases} \dot{z} = \alpha_z(\beta_z(g - y) - z) \\ \dot{y} = z + \frac{\sum_{i=1}^{N} \Phi_i(x)\omega_i}{\sum_{i=1}^{N} \Phi_i(x)} v \end{cases}$$

where ω_i are weights (for the regression) and v and x are auxiliary variable given by the system

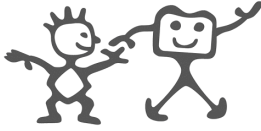$$\begin{cases} \dot{v} = \alpha_v(\beta_v(g - x) - v) \\ \dot{x} = v \end{cases}$$

and

$$\Phi_i = exp\left(-\frac{1}{2\sigma_i^2}(\tilde{x} - c_i)^2\right)$$

where $\tilde{x} = (x - x_0)/(g - x_0)$. Time constants are scaled so to correspond to the duration of the movement to be reproduced. The variable v controls the amplitude of the Gaussian kernel functions; in particular, v is null at the beginning and at the end of the trajectory to ensure that it starts at the right position and converges at the end. Said differently, the variable v ensures that the modification of the trajectory by the Gaussian functions is transient. Note that the kernel functions depend on x which is introduced as a measure of the position in the trajectory, so that they can be adapted to the desired trajectory throughout the trajectory.

More precisely, if y_{des} is the desired trajectory, i.e. the trajectory to be imitated, a weighted regression, with v as input and $u_{\text{des}} = \dot{y} - z \left(= \frac{\sum_{i=1}^{N} \Phi_i \omega_i}{\sum_{i=1}^{N} \Phi_i} v\right)$ as an output, is performed at discrete time steps to minimize the error (i.e. to minimize $J_i = \sum_t (u_{\text{des}}^t - u_i^t)^2$ for each local model $u_i^t = \omega_i v^t$).

Once the trajectory has been learned, it can be easily modulated by modifying the parameters of dynamical systems. For instance, if a particular trajectory is learned for a particular location of

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[9]For clarity reasons, we present here the open loop system. However, in the article, a feedback loop is introduced to deal with external perturbations.

the object, an object at any reachable location can be reached through a trajectory which have the same features as the original one by simply adapting the parameters relatively to the new location. Moreover, in the article a feedback loop on the position error is introduced to deal with external perturbations. Thus, this method allows learning in a sense close to the learning of the a motor program, in the sense that the typical features of the trajectories are learned rather than the exact trajectories.

In conclusion, dynamical systems are a very powerful tool to model human movements, as they combine robustness and adaptability. Indeed, online generation of the trajectories ease their modulation relatively to unpredicted variations, allowing for adaptive movement generation. Moreover, stable, asymptotically attracting solutions of such systems provides robustness against perturbations.

In addition, they are very convenient to model rhythmic and discrete movement as both types of solutions exist naturally in such system. The concept of synchronization is also topical when considering the issue of coordination of movements in single and multi joint systems.

2.2   Optimal control

As we have seen in the introduction, many invariants have been observed in human movements, invariants which are often explained as the result of an optimization process (e.g. minimization of the jerk or of the energy). Thus methods for optimizing trajectories relatively to a given cost function are very interesting for us. We thus introduce here the optimal control approach, that we illustrate with a very interesting approach for simulating discrete and rhythmic movements developed by Biess et al. (2006).

Optimal control is based on the calculus of variations, i.e. problems consisting in choosing a function $\vec{x}(t)$, continuously differentiable on $[x_0, x_1]$, so to

$$\text{minimize:} \quad \mathcal{J} = \int_{t_0}^{t_1} l(t, \vec{x}(t), \dot{\vec{x}}(t)) dt$$
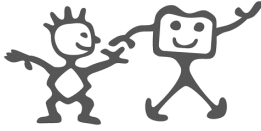$$\text{subject to:} \quad \vec{x}(t_0) = \vec{x}_0,$$

By rewriting this problem under the form of finding a function $\vec{u}(t)$ to

$$\text{minimize:} \quad \int_{t_0}^{t_1} l(t, \vec{x}(t), \vec{u}(t)) dt$$
$$\text{subject to:} \quad \dot{\vec{x}}(t) = \vec{u}(t); \quad \vec{x}(t_0) = \vec{x}_0,$$

we obtain an optimal control problem. More precisely, in optimal control, a control function $\vec{u}(t)$ governs the states of the system ($\vec{x}(t)$) through a set of differential equations. The problem to solve is to find the control function which minimizes a cost function $\mathcal{J}$ which may depend both on the state and the control function, i.e. $\mathcal{J}(x(t), u(t))$. Thus this method is, in its statement, particularly relevant for robotic controllers using dynamical systems.

Some constraints may be added on the terminal states (i.e. constraints of the form $\vec{x}(t_0) = \vec{x}_0$) or the trajectory may be bounded (i.e $x_L \leq \vec{x}(t_0) \leq x_U$), for instance. The major limitation of this approach is that if the dimension of the problem is too high, the existing algorithms fail to converge in most of the cases. Moreover, the existence of an optimal solution can not be ensured in most of the cases.

A method often used to ease the solving of this kind of problem is to discretize the control function, i.e to turn the control variables (and sometimes also the states variable) into functions defined by a

finite number of parameters. For instance, the interval of time can be divided in subintervals on which $u(t)$ is defined as linear or a constant functions ($f(x) = ax + b$ or $f(x) = c$). Thus, instead of optimizing the function $u(t)$, only one or two parameters (a and b or c) need to be optimized. The smaller the time intervals are, the closer the piecewise linear function so obtained will be to the optimal solution.

Another way of proceeding to discretize the problem is to define the function $u(t)$ as a sum of functions on the entire interval. For instance, if the control signal is assumed to be periodic, it can be written as a finite Fourier series. Then, the only parameters to be determined will be the coefficients of the series.

**Simulating discrete and rhythmic multi-joint human arm movements
by optimization of nonlinear performance indices**

Armin Biess, Mark Nagurka and Tamar Flash,
in *Biol Cybern*, 2006

This approach is interesting for mainly two reasons to our point of view:

1. it involves an optimality criterion, i.e. a cost function

2. the difference between discrete and rhythmic movement lies in the constraints on the movements, i.e. in the parametrization of the movement.

The general problem to be solved is expressed in the form

$$\mathcal{J} = \int_0^T F\left(t, \vec{q}(t), \dot{\vec{q}}(t), \ldots, \vec{q}^{(m)}(t)\right) dt$$

subject to:

$$\frac{\partial F}{\partial q_i} - \frac{d}{dt}\frac{\partial F}{\partial \dot{q}_i} + \ldots + (-1)^m \frac{d^m}{dt^m}\frac{\partial F}{\partial q_i^{(m)}} = 0$$
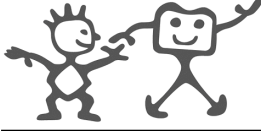
for $i = 1, \ldots, n$, where $\vec{q}$ denotes the generalized coordinates of the n dofs of the system. This constraint correspond to the Euler-Lagrange equations which are a necessary condition for the solution to be a extremum of the cost function $\mathcal{J}$. Note that here, $q(t)$ corresponds to the states function ($x(t)$ above), whereas the first and higher derivatives of $q(t)$ are the control variables ($\vec{u}(t)$).

Some extra boundary conditions are added to specify a discrete or a rhythmic movement. For discrete movement, they are given by

$$\begin{cases} q(0) = q_0 & q(T) = q_f \\ \dot{q}(0) = q_0 & \dot{q}(T) = q_f \\ \vdots & \vdots \\ q^{(m-1)}(0) = q_0 & q^{(m-1)}(T) = q_f \end{cases}$$

and T is the duration of the movement (i.e. the initial and final position is set), whereas for a rhythmic movement they are given by

$$\begin{cases} q(0) = q_0 & q(T) = q_f \\ \dot{q}(0) = \dot{q}(T) & \\ \vdots & \\ q^{(m-1)}(0) = q^{(m-1)}(T) & \end{cases}$$

and T is period of the movement (i.e. after a period of time T the system has to return to its initial conditions, at least for the first derivatives and higher). Note that $q(0) = q(T)$ can also be imposed if needed.

As we mentioned before, a very convenient way to obtain an approximation of the optimal solution is to discretize the control function (which correspond to the states function here). For discrete movement the generalized coordinates are expanded as

$$q_{1,N}(\tau) = \sum_{k=0}^{2m-1} p_{ik}\tau^k + \sum_{k=0}^{N} c_{ik}\phi_k^{(m)}(\tau), \qquad i = 1, \ldots, n$$

where τ is the normalized time $\tau = t/T$. The first terms of the sum correspond to the number of free parameters per dof and the second term is a series of basic functions expressed in terms of Jacobi polynomials, chosen so that the basis is orthogonal.

As for rhythmic movements, as all the derivatives are periodic, it is assumed that the first derivative can be rewritten in terms of a Fourier series, i.e.

$$\dot{q}_{i,N} = \frac{a_{0i}}{2T} + \frac{1}{T}\sum_{k=1}^{N}\left[a_{ik}\cos\left(\frac{2\pi kt}{T}\right) + b_{ik}\sin\left(\frac{2\pi kt}{T}\right)\right]$$

and thus, by integration,

$$q_{i,N} = C_i + \frac{a_{0i}t}{2T} + \frac{1}{T}\sum_{k=1}^{N}\left[\frac{a_{ik}}{2\pi k}\sin\left(\frac{2\pi kt}{T}\right) - \frac{b_{ik}}{2\pi k}\sin\left(\frac{2\pi kt}{T}\right)\right].$$

Using the constraint mentioned before, one gets $C_i = q_{i0}$ and $a_{0i} = 2(q_{if} - q_{i0})$. If the constraint $q(0) = q(T)$ holds, all the generalized coordinates (including $q_{i,N}$) can be written as a Fourier series.

Both series are optimized relatively to the cost function $\mathcal{J}$. This solution, which may not be optimal, is then used as a starting guess to refine the solution by a multiple shooting algorithm[10].

To our opinion, this method of solving an optimal control problem by modeling the control function as a sum of well known functions which parameters are open is a very nice approach. Indeed, this approach is very close to the assumption concerning the modularity of the movement and may be used to reflect the stereotypy in movements. Indeed, the functions in the expansion of the generalized coordinates can be considered as motor primitives.

Furthermore, a possible way of extending this approach to dynamical systems may be to expand the control function in terms of output of dynamical systems. However, a deep study on the stability of such linear combination should be performed first.

In this chapter we have focused on two methods which are particularly relevant for modeling discrete and rhythmic movements, namely dynamical systems and optimal control. Rather than presenting an exhaustive list of the existing models, we have chosen to present some articles representative of the interesting properties of those methods.

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[10]In this method, known criteria of optimality are solved to find the optimal solution. The answer is more precise, but the algorithm often diverges if the initial guess is too far from the solution.

Conclusion

In this review, we have tried to give further insights for the modeling of the human motor system. However, bridging the gap between neurophysiological concepts and mathematical tools is not an easy task. However, it is necessary for neurophysiologists and mathematicians to take the time to overcome differences in approaches and in semantics. Indeed, the motor system is still, to some extent, a black box for us and modeling it requires assumptions to be made and to be expressed explicitly.

We have focused on the question of the generation of discrete and rhythmic movements, as this distinction is mathematically convenient for modeling. Finding the same correspondence in the human motor system turned out to be a difficult task. Indeed, even if the neurophysiological distinction between rhythmic automatisms and voluntary movements seem, at first sight, to be a nice echo of the mathematical distinction, it fails to recreate the diversity observed in human movements.

Furthermore, as experienced by Hogan and Sternad (2007), even classifying human movements into discrete or rhythmic is a challenging task. For us, this difficulty lies in motor variability and in the notion of rhythmicity. Indeed, the key notion in a rhythmic behavior is the repetition, neither of an exact trajectory nor at an exact frequency, but of a specific, recognizable spatial features. What is understood as rhythmic for a human movement is closer to the rhythmicity found in music than in mathematics. In music, the tempo does not lie in every movement, but in the general coherence of the music and of the instrument; analogously, in human behaviors, the rhythmicity lies in the general patterns of the body rather than in the specific trajectories of each joints.

However, this does not mean that modeling human movements by discrete and rhythmic motor primitives is a nonsense. Indeed, combination of both primitives, in the sense of a superimposition - in opposition to a sequencing - opens the possibility of tackling the complexity of human behavior. Via-points rhythmic movements can indeed be seen as discrete - voluntary - corrections of a basic - automatic - rhythmic pattern.

We have presented two mathematical concepts that we think are topical for human movement generation: dynamical systems and optimal control. Indeed dynamical systems are a very powerful tool for modeling adaptive behavior and, moreover, they are particularly well-suited for modeling discrete and rhythmic signals. As for optimal control, we think it is crucial to take in account optimization criteria to model movement behavior, notably because it is a mathematical simple way of reproducing complex behaviors.
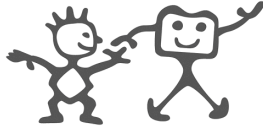
In conclusion, modeling the human motor system through the superimposition of discrete and rhythmic motor primitives seems to be a promising approach. We are currently developing a closed-loop model based on dynamical systems for visually-guided crawling (Righetti and Ijspeert (2006); Degallier et al. (2007)) and we plan to combine it with optimal control. We think that concepts as generalized motor programs, central patterns generators or force fields in humans are also topical for any bio-inspired modeling and we are going to further investigate them.

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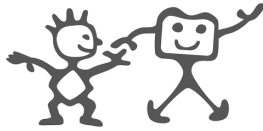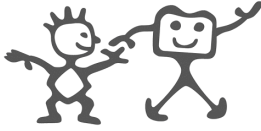
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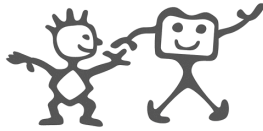
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